Algorithms for Transmission Scheduling in Optical CDMA Networks

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***Abstract*— Transmission scheduling is a media access control mechanism that prevents degradation of throughput in optical CDMA Local Area Networks (LANs) at high offered load. Optical CDMA is a multiple access technique for broadcast optical Local Area Networks. The throughput of an optical CDMA LAN at high offered load is limited by multi-user interference. *Interference Avoidance*, a distributed, contention based media access control mechanism, can prevent throughput degradation at high loads. Interference avoidance consists of *state estimation* and *transmission scheduling*. This work analyzes algorithms for transmission scheduling under perfect state estimation. The analysis shows that transmission scheduling under specific conditions can provide upto 30% network throughput at high offered load. This compares well to non scheduled systems which have close to zero throughput under the same conditions. Simulations show that the performance of transmission scheduling is independent of codeset length and degrades with increase in codeset weight. The results also show that the performance of transmission scheduling does not degrade when used with realistic network traffic based on traffic obtained from a real network.**

KEYWORDS: Networks, Optical communication, Code division multiaccess, Protocols, Access control.

I. Introduction

This work considers a shared medium, packet switched optical CDMA LAN in which several nodes are connected to a passive star coupler to form an all optical broadcast network. Each node on the network is allocated an optical CDMA codeword to receive on. Optical CDMA codewords are sequences of zeroes and ones (unipolar codewords) that are transmitted asynchronously. The codewords are transmitted by binary intensity modulation *i.e.* a one in the codeword is represented by pulse of light. Nodes use ON-OFF keying of the codeword to transmit binary data. To transmit a 1 bit the codeword is sent and to transmit a 0 bit, an all zeros codeword is sent. When a node wants to transmit, it tunes its transmitter to the receiver's codeword and transmits. The code division multiplexing allows several pairs of users to communicate simultaneously.

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The throughput of an optical CDMA LAN is limited by multi-user interference. When several users transmit simultaneously, their packets and hence their codewords overlap. When the optical pulses in the codeword overlap, their optical power is added. Optical pulses from one codeword can be detected by receivers tuned to other codewords. As a result receivers may falsely detect their codewords resulting in packet errors. These false positive errors increase with offered load, resulting in throughput collapse.

Interference Avoidance is a contention media access control mechanism that prevents throughput collapse in optical LANs networks at high offered load. It consists of state estimation and transmission scheduling. State estimation is a mechanism by which nodes on the network estimate the state of the line. Transmission scheduling is a mechanism by which nodes use the estimated state to schedule their transmissions to avoid packet losses due to interference.

The contribution of this paper is the analysis of transmission scheduling algorithms for optical CDMA under perfect state estimation. The analysis quantifies the difference between throughput of systems with and without transmission scheduling. The analysis shows that transmission scheduling under specific conditions can provide upto 30% throughput at high offered load. In contrast, non scheduled systems have close to zero throughput under the same conditions. A sensitivity study of the transmission scheduling algorithms shows that the performance gain depends on certain codeword parameters. The performance is independent of codeword length and degrades with increase in codeword weight. The performance does not degrade with a traffic model based on traffic obtained from a real network.

The paper is organized as follows. Section II provides background on optical CDMA. Section III-A discusses the motivation for Interference Avoidance. Section III-C discusses the channel characteristics of optical CDMA and a representation for the state of the network. Section III-D defines the transmission scheduling algorithms. Section IV analyzes the performance of the transmission scheduling algorithms and Section V discusses a sensitivity study of the algorithms. Section VI discusses the related work in this field. Section VII discusses the conclusions and future work.

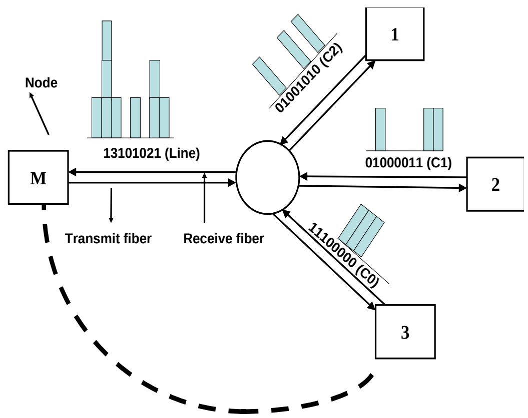

Fig. 1. Typical optical CDMA LAN topology. Nodes are connected by transmit and receive fibers to a passive optical coupler in a star topology.

II. Background

This section provides background on optical CDMA LAN architecture, codeset design and receiver design.

A. Optical CDMA LAN architecture

The optical CDMA network considered in this work is a *shared medium*, *packet switched*, *multiple access* LAN. The physical layer is optical CDMA that uses unipolar encoding and intensity modulation over a single wavelength.

The network consists of several nodes connected by optical fiber to a passive star coupler as shown in Figure 1. The optical coupler consists of several inputs and output ports. Each node is connected to one input and one output port by a transmit and receiver fiber respectively. Signals transmitted on the inputs enter the coupler, merge and are transmitted on all outputs. The star coupler is passive *i.e.* the input power is split equally among the receive fibers and is transmitted to all nodes on the receive fibers. The signal at the output of the coupler on any receive fiber is given by

$$r(t) = (1/K)\sum_{i=1}^{K} s_i(t)$$

where K is the number of ports connected to the coupler and $s_i(t)$ is the signal entering on the i^{th} transmit fiber. The signals on the transmit fibers $s_i(t)$ are binary optical signals and the signal on a receive fiber $r(t)$ is a multilevel optical signal. The signal on the receive fiber may be amplified or attenuated after the coupler.

The network is based on a *Tunable Transmitter-Fixed Receiver (TT-FR)* architecture. A receiver chooses a codeword to receive on and a transmitter which needs to communicate with a receiver tunes to the receiver's codeword. A TT-FR architecture eliminates the need for pre-transmission coordination [1]. The network uses *codeword sharing*. If the number of nodes is greater than the codewords, the codewords are shared among receivers. A higher layer unique identifier such as a link layer address is used to demultiplex packets sharing a codeword. Every node runs a *frame synchronization algorithm* [2] which allows the node to identify that a frame destined for it has arrived and where the first bit of the frame begins.

B. Optical CDMA codeset design

An Optical Orthogonal Codeset (OOC) is a set of (0,1) sequences of length N that satisfies correlation constraints [3]. The term *codeset* is used to refer to the set of sequences, and the term *codeword* is used for a member of the set. Each 0 or 1 of a sequence is called a *chip*, and the codeword represents a data *bit*. For any two codewords in the codeset, the correlation constraints are:

$$\sum_{n=0}^{N-1} s_{(i,n+\tau)}s_{(j,n)} \quad \begin{array}{ll} = w & \text{when } \; i=j, \tau=0 \\ \leq \kappa & \text{otherwise} \end{array}$$

where $s_{(i,n)}$ is the n^{th} chip of the i^{th} codeword, addition is modulo N and $0 \leq \tau \leq N-1$. κ is called the *Maximum Collision Parameter*. The number w of '1 chips' of a codeword of the codeset is called its weight. A particular codeset is specified by the parameters (N, w, κ). The size S of the codeset is the number of codewords in the codeset. Codesets with all codewords having the same weight are called *constant weight* codesets. [3] and [4] describe several codeset construction methods. The codesets used in this work are constant weight codesets generated by the greedy construction method [3]. The rate at which individual chips are transmitted is called the *chipping rate* B. The rate at which the data bits are transmitted is called the *data rate*. The chipping rate is N times the data rate. The codewords are *pseudo-orthogonal* because optical CDMA uses unipolar encoding[1].

C. Optical CDMA receiver design

The optical CDMA receiver (also called a decoder) is a hard-limiting *correlation* receiver [5]. The receiver decodes the codeword in the received signal and regenerates the transmitted data. Figure 2 depicts the operation of a receiver. The input signal from the coupler is a multilevel optical signal. The receiver converts it to a digital optical signal by hard limiting the power in each chip of the received signal. It then decodes the signal to detect a 1 or 0 bit. Let R be the received signal (an N dimensional vector whose components are non-negative integers), and C the codeword being received (an N dimensional vector whose components are binary values). Let $R = [r_0r_1r_2...r_{N-1}]$ and $C = [c_0c_1c_2...c_{N-1}]$. Then the received bit b is given by

$$\begin{aligned} b \quad &= 1 \quad \text{if} \;\; (C \cdot h(R) \geq w) \\ &= 0 \quad \text{otherwise} \end{aligned}$$

where the dot product $\cdot$ of two vectors $[u_0u_1...u_{N-1}] \cdot [v_0v_1...v_{N-1}] = \sum_{i=0}^{N-1} u_iv_i$ and $h()$ is the hardlimiting operation defined as

$$\begin{array}{llll} h(R) = & [s_0s_1...s_i...s_{N-1}] & & \\ \text{where} & s_i = 0 & \text{if} & 0 \leq r_i < 1 \\ & s_i = 1 & \text{if} & r_i \geq 1 \end{array}$$

[1]This contrasts with CDMA on the wireless medium where bipolar encoding is feasible. Bipolar codewords can be designed to be orthogonal.

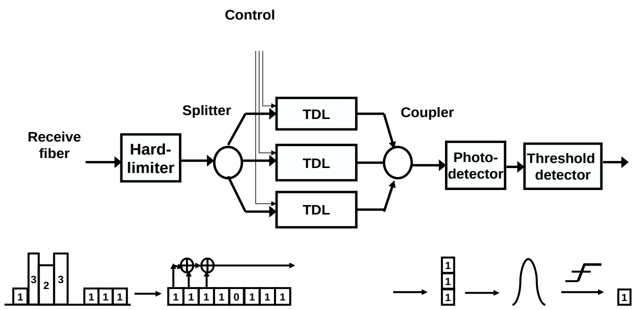

Fig. 2. Optical CDMA receiver: The figure shows a hard-limiting correlation detector that consists of a hard-limiter, decoder, photo-detector and a threshold detector. The receiver is tuned to the codeword 1110000. The power in the 1^{st}, 2^{nd} and 3^{rd} chip positions is summed by the decoder. The photo-detector converts the signal to an electrical signal and the threshold detector detects a 1 bit.

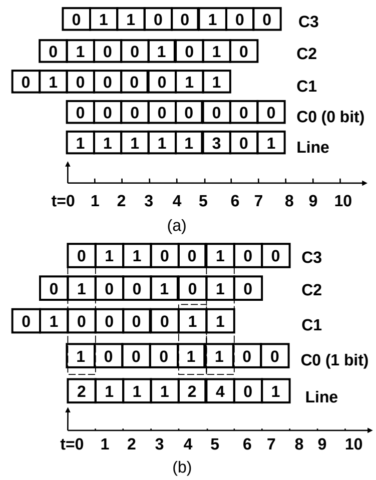

Fig. 3. A bit error. In (a) *C0* is OFF. Codewords *C1* and *C2* have chip overlaps with the 1 chips of *C0*. A false positive error will occur at the receiver. In (b) *C0* is ON, so no false positive error will occur.

III. Interference Avoidance

This section defines the transmission scheduling problem and discusses algorithms for transmission scheduling. First it discusses the the problem of interference, the need for interference avoidance and how it can be implemented as a contention media access control (MAC) protocol. Next, it discusses state and state estimation which are needed for transmission scheduling. Finally, it defines the transmission scheduling problem and discusses the algorithms.

A. The need for Interference Avoidance

Interference occurs due to the multiplexing of packets on a receive fiber. Interference errors increase as the offered load on the network increases. Prior work [6] has shown that without media access control, at high offered load (100%) the throughput of the network approaches zero. The bursty nature of data traffic means that in a operational network there will be periods of overload. *Interference Avoidance* is a distributed, contention based media access control protocol for broadcast, packet based, shared medium optical CDMA Local Area Networks. It improves the throughput of optical CDMA LANs under such conditions.

When two or more packets overlap at a point on a receive fiber (line), the codewords of the packets overlap. Codeword overlaps may cause interference errors at the receiver. When a codeword overlap occurs, two '1 chips' of different codewords may overlap. This is termed a *chip overlap*. An *interference error* will occur during the reception of a codeword if there are enough other codewords on the line which have chip overlaps with the codeword being received. Figure 3 shows codewords from an (8, 3, 3) codeset[2]. The figure is a snapshot of data bits on an optical fiber sent by four nodes. Their combined signal on the line is indicated below the codewords. *C0* is the codeword being received. *C1* and *C2* have 1 chips that overlap with *C0*'s 1 chips. Figure 3(a) shows the case when a '0' data bit is transmitted by the node sending *C0*. Figure 3(b) shows the case when a '1' data bit is transmitted. In (a) the receiver will erroneously detect a codeword (*C0*) because two other codewords overlap with it. The receiver tuned to *C0* will falsely detect a '1' data bit. This results in an error and the loss of the data packet (unless other error correction mechanisms are used). This is an interference error. Therefore, the condition for correct reception of a codeword is that at least one of it's '1 chips' must not have a chip overlap with any other codeword on the line. This works assumes that if an interference error occurs in one bit of a packet, then the entire packet is lost.

A simple example can illustrate the principle behind Interference Avoidance. Consider the codewords shown in Figure 4. The codewords are from a (8, 3, 3) codeset. The signal on the line is called the state of the line (state will be defined formally in Section III-C). If the codeword *C0* is transmitted as shown in Figure 4(a) a false positive error would occur and the packet sent on codeword *C0* would be lost. If it was sent at a different *chip offset i.e.* one chip time later (Figure 4(b)), all three packets could be transmitted correctly. When delayed, codeword *C0* has at least one chip that does not interfere with codewords *C1*, *C2* and *C3*. Hence no false positive can occur and it will be received correctly. Interference Avoidance uses the above principle. A transmitting node estimates the state of the line (*state estimation*) and schedules its packet transmissions to avoid interference errors (*transmission scheduling*).

[2]The figure shows the codewords as chip synchronous. In reality this may not be true. Salehi [5] studied the effect of both chip synchronous and chip asynchronous transmission on a correlation receiver and showed that the chip synchronous case is a upper bound on the BER of the system. Following this result, this work (analysis, simulation and explanations) all assume that the codewords are chip synchronous on the fiber as shown in the figure.

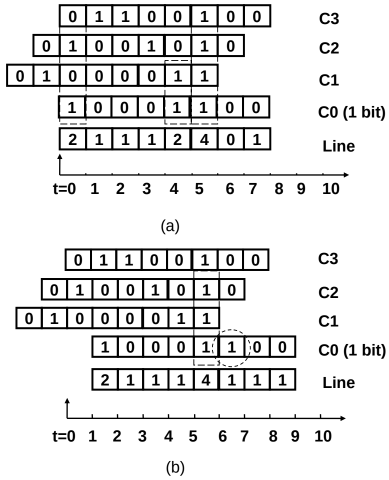

Fig. 4. An interference error is caused in *C0* in (a). No error is caused when the same codewords are sent with a different set of chip offsets. (b) shows codeword *C0* delayed by 1 chip time

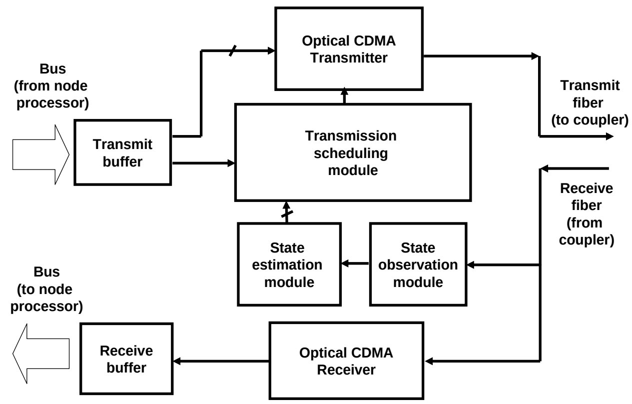

Fig. 5. Block diagram of an Interference Avoidance Network Interface Card.

B. Interference Avoidance media access control

Interference Avoidance is a contention media access control (MAC) protocol. Each node on the network contends for access to the medium using the Interference Avoidance protocol. Figure 5 shows a block digram of an Interference Avoidance Network Interface Card. It consists of an optical CDMA transmitter, optical CDMA receiver, state estimation module and transmission scheduling module (state will be formally defined in Section III-C).

The state estimation module performs two functions: receiving and estimation. It receives the multilevel optical signal on the receive fiber and collects observations of the state of the line. It uses a series of state observations to calculate a state estimate. The state estimation algorithm is always active. It is run continuously in a loop, collecting state observations and calculating a state estimate. The state estimation module consists of both optical and electronic components.

The transmission scheduling module uses the state estimate and the codeword to be used for encoding to calculate a value k (where $0 \leq k < N$) such that interference loss is minimized if the packet's transmission is delayed by k chip times relative to the packet's arrival time. Transmission scheduling is invoked on arrival of a packet from the node processor. It reads the current state estimate and the codeword for encoding and calculates the delay. The optical CDMA transmitter encodes the data and begins transmission after the delay. The transmission scheduling module is purely electronic and must compute the transmission delay within a few bit times of the current packet's arrival.

The electronic part of the state estimation module and the transmission scheduling module may be integrated and implemented in a single ASIC chip and optimized for minimum latency. This paper focuses on the analysis of the performance of a network of Interference Avoidance nodes. Future work will examine the implementation of the NIC hardware.

C. Optical CDMA State and State Estimation

The signal at any time at any point on the receive fiber of an optical CDMA LAN is a multilevel signal due to the sum of the codewords. The *state* of the line is a vector of length N equal to the sum of the codewords at output of the coupler assuming that all nodes are transmitting 1 bits.

$$S(t) = [s_0 s_1 s_2 .... s_{N-1}] = \sum_{i=0}^{M} rot(C_i, \phi_i)$$

where M is the number of codewords on the line at the output of the coupler at time t, $C_i = [c_0 c_1 ... c_{N-1}]$ is a codeword present at the output of the coupler, $rot(C_i, \phi_i)$ is a vector of length N equal to the left rotation of the codeword C_i by ϕ_i and ϕ_i is the number of chips between C_i's leading chip (*i.e.* the chip that was transmitted first, c_0) and the output of the coupler. It is a hypothetical, idealized representation of the state of the system. It is possible that the state may never actually be observed as a signal on the optical fiber. The state transitions to a new value on a packet arrival or departure.

A node can receive the multilevel optical signal and convert it to electronic form using hardware. The received data can be used to construct a series of state observations.

The state estimation problem is to calculate the state of the line given a series of state observations collected from the receive fiber. A simple state estimation algorithm could estimate the state by averaging the state observations and dividing the result by the probability that a codeword is transmitting a 1 bit. An appropriate higher layer encoding format (such as 8B/10B) may be used to ensure that the observations have a sufficient number of codewords transmitting a 1 bit.

This work assumes *perfect state estimation i.e.* all the nodes on the network know the state of the line. Future work will examine the impact of realistic state estimation.

D. Transmission scheduling

Transmission scheduling is a process by which a node, given a state estimate and a codeword to be transmitted, calculates a codeword delay such that interference errors are reduced. If

$c_{tx} \leftarrow$ Codeword to be transmitted
$state \leftarrow$ State estimate
$hstate \leftarrow hardlimit(state)$
$t_d \leftarrow 0$
for *offset* $= 0$ to N
if ($hstate$ & $c_{tx} \neq c_{tx}$) then
mark *offset* as a feasible offset
rotate c_{tx} to the right by one chip
$t_d \leftarrow$ any feasible offset

TABLE I

THE PURE SELFISH TRANSMISSION SCHEDULING ALGORITHM

$c_{tx} \leftarrow$ Codeword to be transmitted
$state \leftarrow$ State estimate
$hstate \leftarrow hardlimit(state)$
$t_d \leftarrow 0$
for *offset* $= 0$ to N
if ($hstate$ & $c_{tx} \neq c_{tx}$) then
$newstate = state + c_{tx}$
$numoverlaps = overlaps(newstate)$
if ($numoverlaps < threshold$)
mark *offset* as a feasible offset
rotate c_{tx} to the right by one chip
$t_d \leftarrow$ any feasible offset

TABLE II

THE THRESHOLD TRANSMISSION SCHEDULING ALGORITHM

transmission can be scheduled, then the scheduling algorithm returns an *offset* k such that $0 \leq k < N$. The offset is the number of chips that the packet transmission should be delayed. The offset is measured with respect to the estimated state of the line. If transmission is not possible, then the packet transmission is deferred by returning it to a higher layer for a retransmission attempt. Other defer mechanisms such as 1, non and p-persistent sensing [7] may also be used. The transmitting node does not have a receiver to detect errors in its transmitted packet during transmission. Therefore packets which experience interference errors during transmission are transmitted until completion. Transmission scheduling is done on a per packet basis.

This work assumes *perfect state estimation* by all nodes on the network. In perfect state estimation, every node knows the state of the line. All nodes see the same state at the same time. The network is assumed to have zero propagation delay. It is also assumed that state estimation is instantaneous and there is no delay between state estimation and transmission scheduling. The transmitter knows its distance from the coupler ($a = 0$), therefore it can schedule its transmission using the state estimate. This an idealized, unrealizable state estimation algorithm which allows easy analysis of transmission scheduling.

When a packet is transmitted, interference errors could be caused in itself or in other packets. One of four possible events (*transmission events*) could occur:

- *Preserves self/Destroys other*: The packet may be received without error (*i.e.* preserves itself), but may cause an error in one or more packets (*i.e.* destroys others) on the line.
- *Preserves self/Preserves others*: The packet preserves itself and also preserves all other packets on the line.
- *Destroys self/Preserves others*: The packet destroys itself but preserves all other packets on the line.
- *Destroys self/Destroys others*: The packet destroys itself and destroys one or more other packets on the line.

When scheduling a transmission, any one of the four transmission events could happen. A *transmission scheduling strategy* is a subset of the transmission events that a node tries to achieve. A *transmission scheduling algorithm* is an implementation of a strategy. Two possible transmission scheduling strategies are *selfish* and *cooperative*. An algorithm follows a *selfish strategy* if the node schedules its packet transmission only if the packet either *Preserves self/Preserves others* or *Preserves self/Destroys others*. An algorithm follows a *cooperative strategy* if the node schedules its transmission only if it *Preserves self/Preserves others* or *Destroys self/Preserves others*. Simple implementations of cooperative strategy are either not feasible for all codesets or result in low throughput. A *pseudo-cooperative* strategy attempts to reduce the probability of destroying other packets on the line. It is a best effort strategy which increases the probability of the events *Preserves self/Preserves others* or *Destroys self/Preserves others*.

The following sections discuss three transmission scheduling algorithms: *Pure selfish*, *Threshold* and *Overlap section scheduling*. The transmission scheduling algorithms implement either selfish or pseudo-cooperative strategies or both. The section compares their performance to Aloha-CDMA *i.e.* optical CDMA without any media access control.

1) Pure selfish scheduling: The pure selfish algorithm schedules a packet transmission only if the state of the line permits transmission without loss of its own packet. The algorithm is specified in Table I. The algorithm searches for chip offsets where there at least one of the '1 chips' from the codeword to be transmitted aligns with a '0 chip' in the state vector, thus 'selfishly' ensuring correct reception of this codeword. It chooses one of these offsets at random.

2) Threshold scheduling: The threshold scheduling algorithm searches for chip offsets where at least one of the '1 chips' from the codeword to be transmitted aligns with a '0 chip' in the state vector and the number of chip overlaps in the resulting state is below a threshold. It chooses one of these offsets at random. The threshold is expressed as a fraction of the codeword length N, called the *threshold parameter* α. The algorithm is specified in Table II.

3) Overlap section scheduling: The overlap scheduling algorithm searches for chip offsets where at least one of the '1 chips' from the codeword to be transmitted aligns with a '0 chip' in the state vector and the number of chip overlaps in the resulting state is below a threshold. It chooses one of these offsets at random. The algorithm is specified in Table III.

IV. PERFORMANCE STUDY

In this section the transmission scheduling algorithms are analyzed and simulated. A mathematical analysis is described which shows that the algorithms prevent throughput degradation. It is also shown that the threshold and overlap section algorithms have lower packet errors compared to the pure

$c_{tx} \leftarrow$ Codeword to be transmitted
$state \leftarrow$ State estimate
$hstate \leftarrow hardlimit(state)$
$t_d \leftarrow 0$
for *offset* $= 0$ to N
  if ($hstate$ & $c_{tx} \neq c_{tx}$) then
    $newstate = state + c_{tx}$
    $numoverlaps = overlaps(newstate)$
    $numones = ones(newstate)$
    if ($numoverlaps < numones$)
      mark *offset* as a feasible offset
  rotate c_{tx} to the right by one chip
$t_d \leftarrow$ any feasible offset

TABLE III
THE OVERLAP SECTION TRANSMISSION SCHEDULING ALGORITHM

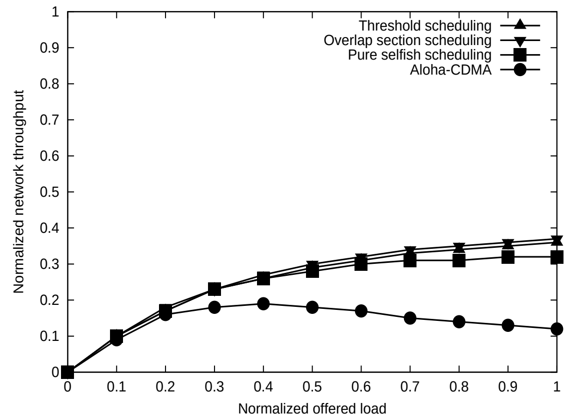

Fig. 6. Comparison of the performance of the transmission scheduling algorithms based on analysis. The traffic model is Poisson arrivals with exponentially distributed packet lengths. The codeset is $(10, 3, 3)$ and codewords are chosen uniform randomly. For the threshold scheduling algorithm, the threshold parameter was set to 0.5

selfish algorithm. A simulation study is used to validate the mathematical analysis.

The metric used to evaluate performance is the normalized network throughput at different values of the normalized offered load. The *normalized offered load* is the arrival rate (in packets/s) expressed as a fraction of the maximum possible arrival rate (in packets/s) of the network when it is used as a single channel network[3]. The arrival rate is defined as the aggregate rate at which packets arrive to all the nodes for transmission on the network. The *normalized network throughput* is the ratio of the number of packets that are transmitted over the network without error to the total number of packets offered for transmission multiplied by the normalized offered load. It is a measure of the throughput of packets transmitted without error at a particular offered load. Appendix I defines the metrics formally and derives expressions for them.

A. Analysis

This section describes a mathematical analysis of the transmission scheduling algorithms. The analysis follows the steps below:

- First, an expression for the normalized network throughput is derived in terms of the aggregate arrival rate. This expression is then expressed in terms of the number of codewords on the line (*i.e.* at a point on the receive fiber) and the probability of packet error (packet error rate) (Appendix I).
- Then, a concise representation of line state which allows easy mathematical manipulation is defined (Appendix II-A).
- Using this state representation, expressions are derived for the number of codewords at a point on the line and the probability of packet error when the system is in any state (Appendix II-B and II-C).
- Based on the transmission scheduling algorithm (Aloha-CDMA, Pure Selfish, Threshold, Overlap section) the state transition probabilities are calculated and a state transition diagram is constructed. Under the assumptions of Poisson arrivals and exponentially distributed packet sizes, the state transition diagram can be viewed as a Markov chain. The Markov chain is solved for equilibrium state probabilities at a particular offered load (Appendix III).

The analysis can be used to determine the normalized throughput at any normalized offered load. A graph of the normalized throughput vs. normalized offered load for different scheduling algorithms is shown in Figure 6. The traffic model is Poisson arrivals and exponentially distributed packet sizes. Codeword allocation is uniform random over the codeset. The graph indicates that a system with no transmission scheduling (Aloha-CDMA) suffers throughput degradation. This may be seen from the performance at high offered loads. Beyond an offered load of around 0.5, the network throughput decreases and tends to zero at high offered loads. In contrast, the transmission scheduling algorithms all prevent throughput degradation. Throughput is stabilized at around 30% of the maximum throughput and remains stable as offered load is increased.

B. Simulation

A discrete event based packet simulator was designed to validate the mathematical analysis. The simulator modeled multiple nodes on a broadcast shared medium optical CDMA LAN. It implemented different state estimation algorithms, transmission scheduling algorithms and a hard-limiting correlation receiver. Unless specified otherwise, the default parameters for the simulations are specified in Table IV. The optical orthogonal codeset construction method was the greedy construction method [3]. It was used to generate several codesets for a given set of codeset parameters. The results in this work did not depend on the codeset or the algorithm used to generate the specific optical orthogonal codeset. The results from the simulations are the mean of around 10 runs of anywhere from 10,000 to 100,000 packets each and standard deviations are shown on the graphs.

[3]For an optical CDMA network of chipping rate B chip/s, the maximum possible data rate of the network when used as a single channel network is B b/s (the chipping rate becomes the bit rate). The maximum possible arrival rate in packets/s is B divided by the average packet size in bits.

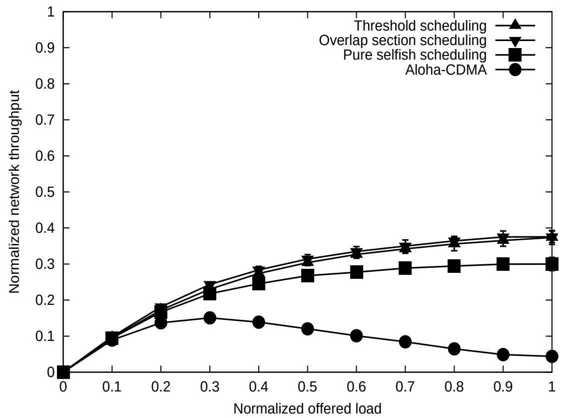

Fig. 7. Comparison of the performance of the transmission scheduling algorithms based on simulation. The traffic model is Poisson arrivals with exponentially distributed packet lengths. The codeset is $(10, 3, 3)$ and codewords are chosen uniform randomly. For the threshold scheduling algorithm, the threshold parameter was set to 0.5

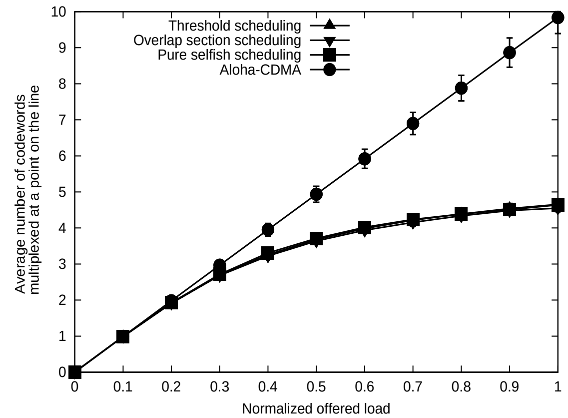

Fig. 8. Comparison of the codeword multiplexing of the different transmission scheduling algorithms based on simulation. The traffic model is Poisson arrivals with exponentially distributed packet lengths. The codeset is $(10, 3, 3)$ and codewords are chosen uniform randomly. For the threshold scheduling algorithm, the threshold parameter was set to 0.5

Figure 7 shows the results of simulation for the same codeset as described in the analytical results. The results are quite similar to the analytical results. All transmission scheduling algorithms prevent throughput degradation. Also the overlap section and threshold scheduling show marginally higher throughput than pure selfish scheduling. The analytical model over predicts the throughput for Aloha-CDMA. This is because the analysis is based on a finite state model. A finite state model is suitable for transmission scheduling algorithms which limit the traffic on the line. However for Aloha-CDMA, the finite state model over predicts the equilibrium state probabilities. As a result the analytical results differ from simulation.

Though Figures 6 and 7 indicate that all three scheduling algorithms has approximately the same throughput, the algorithms differ in the packet error rate. Figures 8 and 9 show the average number of codewords multiplexed on the line at a point on a receive fiber and the average packet error rate for the transmission scheduling algorithms.

There is a trade-off between the number of codewords at a point on the line and the packet error rate. The transmission scheduling algorithm tells a node *if* it can transmit. It also tells the node *when* to transmit. By doing this the transmission scheduling algorithm controls two quantities:

- The number of codewords at a point on the line.
- The chip offsets of the codewords on the line which affects the packet error rate.

The trade-off can be understood by considering the performance of Aloha-CDMA. As the offered load increases, the number of codewords on the line for Aloha-CDMA shows a linear increase with offered load. Initially as the number of codewords on the line increases, the network throughput increases. But as the number of codewords on the line increases further, interference errors increase and as a result the packet error rate increases. As a result the network throughput falls. Therefore as the offered load increases, the network throughput attains a maximum and then decreases. To the left of the maximum, the throughput is lower due to the low number of codewords on the line. To the right of the maximum, the throughput is lower due to the higher packet error rate. The objective of the transmission scheduling algorithm is to keep the system operating point close to the maximum irrespective of the offered load. Therefore, if the transmission scheduling algorithm is aggressive and allows more codewords on the line, the packet error rate increases, lowering throughput. On the other hand if the algorithm is conservative and does not allow enough codewords on the line, the throughput remains low. So the transmission scheduling algorithm must carefully balance the number of codewords on the line and the codeword offsets so as to maximize throughput.

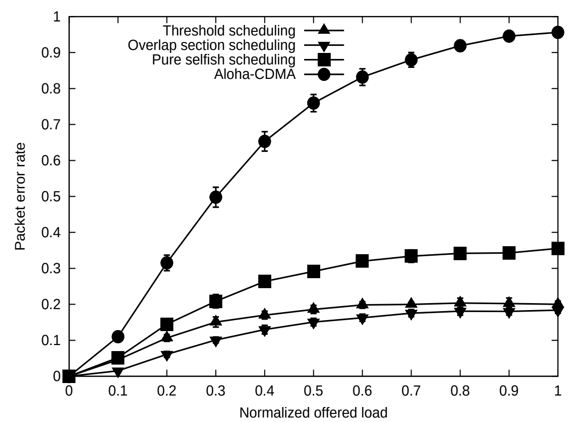

Fig. 9. Comparison of the packet error rate for different transmission scheduling algorithms based on simulation. The traffic model is Poisson arrivals with exponentially distributed packet lengths. The codeset is $(10, 3, 3)$ and codewords are chosen uniform randomly. For the threshold scheduling algorithm, the threshold parameter was set to 0.5

The difference between the three scheduling algorithms is evident from Figure 9. There are differences in the packet error rate. The threshold and overlap section scheduling algorithms are conservative and constrain the number of overlapping chips. This results in a lower number of codewords on the line and low errors due to interference. However the selfish algorithm is aggressive and admits a larger number of

Parameter	Default value
Codeset parameters:	
Codeset length N	100
Number of wavelengths Λ	1
Codeset weight w	3
Maximum crosscorrelation parameter κ	3
Size of codeset S	100
Chipping rate:	10 Gc/s
Codeword allocation:	Uniform random
Interference Avoidance parameters:	
Transmission scheduling algorithm:	Threshold scheduling
Threshold:	0.5
State estimation algorithm:	Perfect state estimation
Traffic parameters:	
Inter-arrival time distribution	Exponential
Normalized offered load	1
Packet size distribution	Exponential
Average packet size	1000 $bytes$
Destination address distribution:	Uniform random
Topology parameters:	
Node to coupler distance distribution	Deterministic
Average node to coupler distance	0 m
Number of nodes	100

TABLE IV
PARAMETER LIST AND DEFAULT VALUES FOR THE TRANSMISSION SCHEDULING SENSITIVITY STUDY.

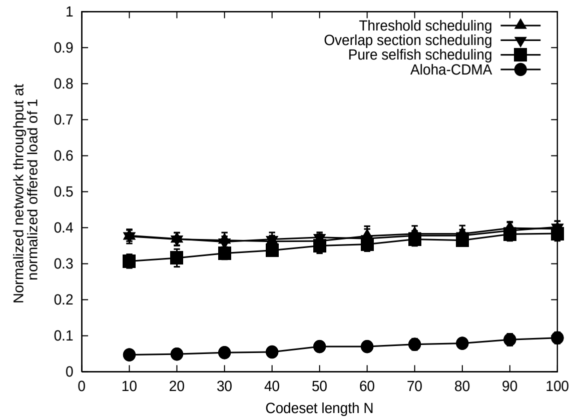

Fig. 10. Comparison of the performance of the transmission scheduling algorithms as the codeset length is varied (based on simulation). The traffic model is Poisson arrivals with exponentially distributed packet lengths. The codeset weight is 3 and $\kappa = 3$. Codewords are chosen uniform randomly from the set. For the threshold scheduling algorithm, the threshold parameter was set to 0.5

codewords with a higher probability of packet error. When packets are lost due to interference errors, the higher layers of the protocol stack must recover through some form of Automatic Repeat Request (ARQ) or Forward Error Correction (FEC). Therefore the threshold and overlap section scheduling algorithms are better choices because a larger fraction of the packets transmitted are transmitted without error.

V. SENSITIVITY ANALYSIS

A simulation based study was conducted to allow a deeper sensitivity analysis of the transmission scheduling algorithms. The study considered parameters at the physical layer (codeset parameters), the media access control layers (the scheduling algorithm parameters) and the traffic model (packet arrival and size distributions). The objective of this study is to quantify the impact of these factors on the transmission scheduling algorithms. The sensitivity analysis consisted of quantifying the:

- Effect of varying the codeset length.
- Effect of varying the codeset weight.
- Effect of different packet size distributions.
- Performance under realistic network traffic.

A. Effect of varying the length of the codeset

Increasing the length of the codeset has several effects. As N increases, the scheduling algorithm can schedule a larger number of codewords simultaneously on the line. Therefore more nodes can transmit in parallel without error. However the nodes transmit at a lower data rate. The results show that the two effects balance each other and the network throughput is constant when N is varied. Figure 10 shows a graph of network throughput at an offered load of 1 vs. the codeset length. Therefore, network throughput is independent of codeset length and transmission scheduling scales with increase in codeset length.

B. Effect of varying the weight of the codeset

Changing the codeset weight w has two effects: As w is increased, the threshold on the correlation receiver can be increased. Increasing the threshold makes the codewords more resistant to interference. This is because it takes a larger number of chip overlaps to cause a false positive error. However, with the increase in weight, each codeword causes more interference with other codewords. An increase in w also makes it difficult for the transmission scheduling algorithm to schedule codewords on the line without causing interference errors. This reduces the number of codewords that can be simultaneously transmitted on the line. The results show that as the weight increases the throughput decreases rapidly. The reduction in the codewords on the line and the increased interference offset any gains in the resistance to interference. It has been shown [8] that codesets with higher weight have better bit error rate characteristics at low loads. At low loads the increase in resistance to interference dominates resulting in higher throughput for high weight codesets. However as the load is increased, the effect of interference tends to dominate, resulting in lower throughput for high weight codesets. Figure 11 shows a graph of the network throughput at offered load of 1 vs. the weight.

C. Effect of packet size distribution

Figure 12 shows the packet throughput as the average packet size is varied. The traffic model is Poisson arrivals and exponentially distributed packet sizes. The figure shows that the packet size has no effect on packet throughput. However, studies indicate that real network traffic packet size distributions may not be exponential [9]. Recent packet statistics obtained from a backbone network [10] exhibit a trimodal distribution. In one trace, about 70% of the packet

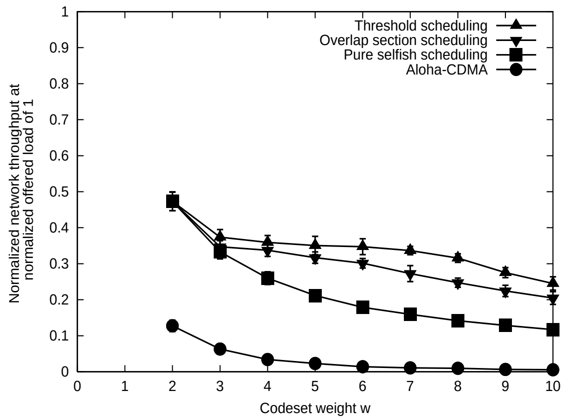

Fig. 11. Comparison of the performance of the transmission scheduling algorithms as the codeset weight is varied (based on simulation). The traffic model is Poisson arrivals with exponentially distributed packet lengths. The codeset length is 100 and $\kappa = 3$. Codewords are chosen uniform randomly for the codeset. For the threshold scheduling algorithm, the threshold parameter was set to 0.5

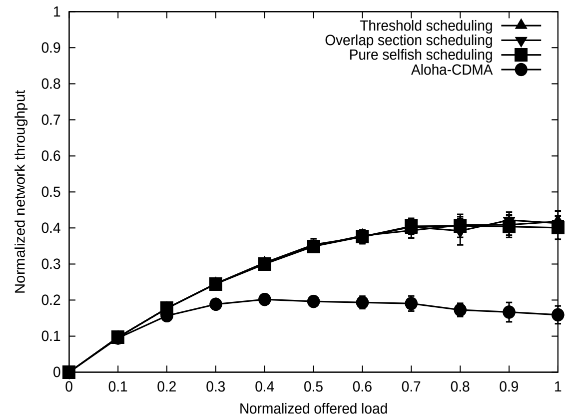

Fig. 13. Comparison of the performance of the transmission scheduling algorithms for a trimodal packet size distribution (based on simulation). The traffic model is Poisson arrivals with packet size distribution consisting of 70% 40 byte packets, 20% 1500 byte packets and 10% 500 byte packets. The codeset is $(100, 3, 3)$ and codewords are chosen uniform randomly. For the threshold scheduling algorithm, the threshold parameter was set to 0.5

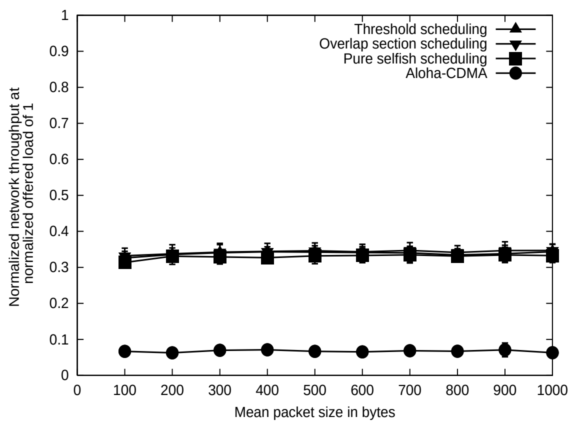

Fig. 12. Comparison of the performance of the transmission scheduling algorithms as the average packet length is varied (based on simulation). The traffic model is Poisson arrivals and exponentially distributed packet sizes. The codeset is $(100, 3, 3)$ and codewords are chosen uniform randomly. The algorithm was threshold scheduling, the threshold parameter was set to 0.5

sizes were 40 bytes, about 20% were 1500 bytes and the remaining were around 500 bytes long. A traffic model with such a trimodal packet size distribution was used to drive a simulation that used Poisson arrivals, uniform random codeset allocation and a (100, 3, 3) codeset. The results are shown in Figure 13. The graph shows interesting behavior. Aloha-CDMA does not degrade as much as in the case of Poisson traffic/exponential packet sizes. The other transmission scheduling algorithms have almost 25% higher throughput when compared to performance with exponentially distributed packet sizes. A majority of the packets are small size packets (40 bytes). Study indicates that the packet error rate is lower for short packets than for long packets. Fewer long packets are transmitted on the line and a large fraction of them are lost due to errors. This *squeeze through effect* results in an increase in the aggregate network packet throughput. This behavior occurs when the fraction of shorter packets is fairly high (60-70%). This is similar to behavior on wireless links where short packets tend to experience lower error rates than long packets [11].

The squeeze through effect can be demonstrated analytically and through simulation for a network with a bimodal distribution of packet sizes which uses a pure selfish transmission scheduling algorithm and a codeset with $\kappa = w$. Consider a network where the traffic has two packet types of sizes l_1 and l_2 where $l_1 < l_2$. Let the fraction of packets of size l_1 be γ. Then the average packet size on the network is $l_{avg} = (\gamma)l_1 + (1-\gamma)l_2$. The throughput of such a network can be calculated by finding the probability of packet error P_{error} and the number of codewords on the line N_l.

The average number of codewords on the line at any point on the receive fiber N_l depends on the transmission scheduling algorithm. If there are N_l codewords on the line, then $N_l w$ 1 chips were added to the state. Of these, $N_l + (w-1)$ 1 chips were aligned with 0 chips of the state (during selfish scheduling) and $(N_l - 1)(w-1)$ 1 chips were added randomly to any position ($\kappa = w$). The probability that the state vector has no 0 chips is

$$P_{full} = 1 - (1 - \frac{1}{(N - (N_l + (w-1)))})^{(N_l-1)(w-1)}$$

At an offered load of 1, packets are arriving for transmission at a rate much higher than the rate at which packets are transmitted (the transmission scheduling does not allow all packets to be transmitted). When a packet departs from the line, a few chips of the state may change from 1 to 0. The next packet arrival will result in a transmission of a packet such that the 0 chips will be filled. Therefore under equilibrium conditions, for the pure selfish scheduling algorithm, P_{full} will be close to 1. Through simulation P_{full} is determined to be around 0.85 for the pure selfish algorithm under Poisson arrivals and exponentially distributed packet sizes. It is assumed that this is true for bimodal packet distributions too. Therefore, the number of codewords on the line can be calculated by finding N_l such that P_{full} is close to 1.

Interference errors in a packet on the line are caused

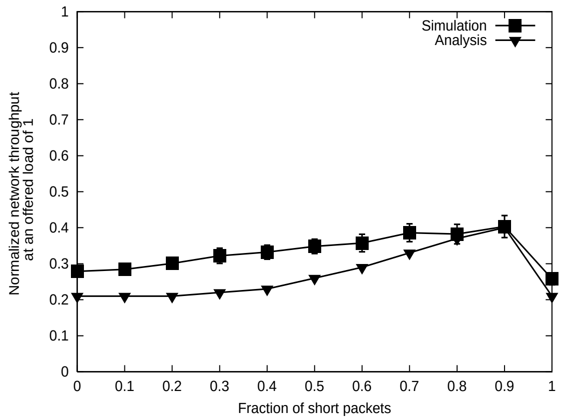

Fig. 14. The squeeze through effect. The throughput is maximized when the fraction of short packets is 0.9. The graph shows both analytical and simulation results. The transmission scheduling algorithm is pure selfish. The packet sizes are 50 bytes and 1000 bytes. The codeset is (100,3,3).

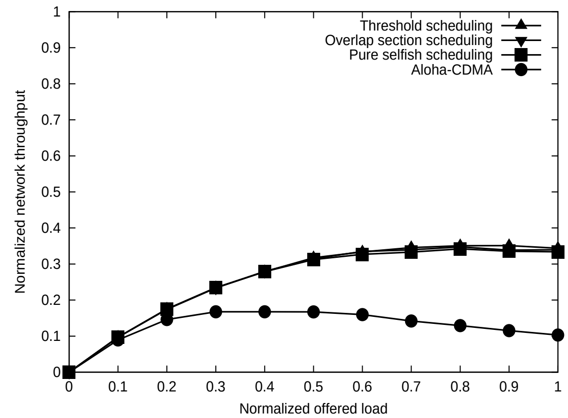

Fig. 15. Comparison of the performance of the transmission scheduling algorithms for a realistic traffic model (based on simulation). The traffic model was based on real network traffic traces (see description). The codeset is $(100, 3, 3)$ and codewords are allocated to addresses. For the threshold scheduling algorithm, the threshold parameter was set to 0.5

by packets that arrive during the packet's transmission. The transmission scheduling algorithm allows only a fraction of the arriving packets (called colliding packets) to be transmitted. The probability of packet error in a codeword on the line is the probability that at least one of its colliding packets causes an interference error. If the number of colliding packets is n_c, then n_c 1 chips are added to the state selfishly (align with 0 chips) and $n_c(w-1)$ 1 chips are added to the state in random positions. These random positions are chosen from $N-1$ possible choices (1 chip is chosen selfishly). The probability that one of the added 1 chips overlaps with the one of the 1 chips of the codeword on the line is $p = w/(N-1)$. The probability of packet error is the probability that more than $w-1$ overlaps occur. Therefore,

$$P_{error} = 1 - \sum_{k=0}^{w-1} \binom{n_c(w-1)}{k} p^k (1-p)^{n_c(w-1)-k}$$

At a normalized offered load of 1, the average packet inter-arrival time is $t_{arrival} = l_{avg}/B$ where B is the chipping rate of the network. The transmission time for packets of type 1 is $t_1 = l_1/(B/N)$ where N is the codeset length. The average number of colliding packets for packet type 1 is,

$$n_{c1} = (t_1/t_{arrival}) * (N_l/N)$$

A similar expression can be derived for n_{c2}. This can be used to calculate the probability of packet error for each type of packet P_{error1} and P_{error2}.

The normalized network throughput based on the definition in Appendix I is given by

$$Th = (N_l/N)(\gamma(1-P_{error1}) + (1-\gamma)(1-P_{error2}))$$

A graph of normalized network throughput vs. the fraction of short packets is shown in Figure 14. The packet sizes were set to 50 bytes and 1000 bytes. The results (both analysis and simulation) show that the normalized network throughput peaks at a particular value of the fraction of small packets confirming the squeeze through effect. The long packets experience high packet error rates and the short packets experience low error rates. The throughput attains a maximum when the fraction of short packets reaches a particular value (around 0.9).

The higher throughput of shorter packets may not be a desirable characteristic, because it is unfair to longer packets. Future work will address the issue of providing a uniform dropping probability to all traffic. Possible alternatives include using constant packet sizes or varying the codeset length.

D. Performance with real network traffic

The assumption of Poisson arrival and exponential distributed packets lengths is convenient for analysis. However, the inter-arrival and packet size distribution of real network traffic could be different depending on when and where it is observed.

Simulations were performed with traffic traces obtained from a real network link to understand the impact of real packet arrival times. Traffic traces from a single OC48 [12] link were used. Several of these traces were merged to generate traffic of different offered loads. The packet sizes, source addresses, and destination addresses were preserved during merging. The packet size distribution was trimodal (35% of the packets were 40 bytes, 30% were 500 bytes and 35% were 1500 bytes). The traffic file had approximately 6000 unique source addresses and 40000 unique destination addresses. In contrast to all the previously described results, in this case the nodes mapped destination addresses to codewords before transmission. Where codewords were insufficient, codeword reuse was used.

The results of the simulation are shown in Figure 15. The results indicate that the performance is similar to that of the Poisson arrivals/exponentially distributed model, indicating that it was a fairly reasonable choice for analysis. Note that the performance improvements of the previous section due to the squeeze through effect are not visible here. The proportion of smaller packets (40 bytes) in the traffic was not sufficient to cause the squeeze through effect.

VI. RELATED WORK

Work related to Interference Avoidance can be divided into four areas: Bit error rate analysis of optical CDMA networks, optical CDMA codeset design, FEC for optical CDMA and media access control in optical/wireless networks.

Salehi [5], [8] analyzed an optical CDMA based network and developed expressions for the bit error rate of a network that uses codesets with $\kappa = 1$. The analysis also determined the bit error rate for codesets with different lengths and weights and with hard-limiting at low loads using Aloha-CDMA. This work examines these results in the context of transmission scheduling at high offered loads.

The area of optical CDMA code design has focused on construction of codesets with large size. Chung et al. [3] described several algorithms to construct OOCs. These constructions are for codes with maximum crosscorrelation parameter $\kappa = 1$. Chung and Kumar [13] described a method for construction of codes with $\kappa = 2$. Several construction methods for OOCs are described in [4] and [14] among others.

Efforts at reducing packet errors in optical CDMA have mostly focused on using error correcting codes on top of optical CDMA. Hsu et al. [15] analyzed the performance of slotted and unslotted optical CDMA packet networks. They developed expressions for the throughput of the network and showed performance can be improved using Forward Error Correction (FEC) codes and hard limiters. Muckenheim et al. [16] studied the effect of bit error probability on the packet error probability and suggested the use of block codes to reduce packet errors. They also described a random delay protocol to reduce the errors incurred during periods of high activity and showed throughput improvement. The mechanism detects periods of high activity and defers transmissions. In contrast Interference Avoidance does not use any FEC and schedules packet transmissions to avoid interference.

Contention media access control mechanisms such as Carrier Sense and Collision Detection for optical networks have been studied in [17]. Interference Avoidance in optical CDMA is analogous to carrier sensing in single channel shared medium networks. Interference Avoidance can be viewed as the result of applying CSMA design principles to the optical CDMA physical layer.

Synchronous-CDMA [18] is a CDMA based physical layer for cable networks. Nodes reserve codewords to use in a time slot through a centralized controller. This mechanism is similar to the Interference Avoidance techniques in that it performs admission control and restricts the number of simultaneous users on the network. However it is centralized and admission control is static (by codeword assignment).

In wireless networks, several mechanisms have been proposed to take advantage of knowledge about channel conditions. State in wireless networks has to account for the channel noise and multi-path effects. Channel load sensing [19], [20], [21], MIMO/CSIT based media access control [22] and scheduling algorithms [23] are systems where wireless state has been used to schedule transmissions. The differences between these mechanisms and Interference Avoidance are the state description and the scheduling algorithms. In these systems state is a scalar variable and media access control is through admission control *i.e.* the number of simultaneous users is controlled.

VII. CONCLUSIONS AND FUTURE WORK

This work has presented an analysis of transmission scheduling algorithms for optical CDMA media access control. The analysis quantified the difference between throughput of systems with and without transmission scheduling and showed that transmission scheduling achieved 30% throughput while non scheduled systems had close to zero throughput. Simulations showed that the throughput of transmission scheduling is independent of codeset length. It also showed that an increase in weight can lead to a degradation in the performance of these algorithms, although the degradation is not as bad as systems without transmission scheduling. Simulations also showed that transmission scheduling prevents degradation when used with a realistic traffic model based on traffic obtained from a real network.

Limitations of this work include the fact that it assumes perfect state estimation and neglects errors due to synchronization and receiver contention. Future work will explore the impact of realistic state estimation.

Work in progress includes a testbed implementation of the transmission scheduling hardware. The testbed demonstrates a simplified form of threshold transmission scheduling by transmitting bits such that the number of chip overlaps is constrained. Measurements indicate that the bit error rate is substantially lower for this system than for a system without transmission scheduling.

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APPENDIX I
OPTICAL CDMA LAN PERFORMANCE METRICS

Let the aggregate arrival rate of packets to the network be λ packets/s. If the average packet size is L bytes, the chipping rate is B chips/s, then the maximum possible packet arrival rate on a single channel network is $B/8L$ packets/s. Therefore, the normalized offered load ρ is,

$$\rho = 8L\lambda/B$$

Consider an optical CDMA LAN where the arrival rate of packets is λ packets/s. The service rate μ (in packets/s) of the network is the rate at which packets are transmitted on the network. For an optical CDMA LAN using a codeset (N, w, κ), an average packet size of L bytes, and a chipping rate B chips/s the service rate is $\mu = B/(8LN)$ packets/s. Substituting for B in the expression above,

$$\rho = 8L\lambda/(8LN\mu)) = \lambda/\mu N$$

Consider an Aloha-CDMA system where packets are transmitted on arrival. Consider any point on a receive fiber. If each packet takes an average time of $1/\mu$ seconds at that point, then Little's law, the average number of packets at that point (on the line) is λ/μ.

For any other transmission scheduling algorithm, the same average number of packets are offered for transmission. However the transmission scheduling algorithm does not allow all of these packets to be transmitted. Let the average number of packets on the line at any point of a receive fiber be N_{online}, of which a fraction P_e are lost due to error. The ratio of the average number of error free packets transmitted to the average number of packets offered for transmission is $N_{online}(1 - P_e)/(\lambda/\mu)$

Therefore,

$$\begin{aligned} Th_{norm} &= (N_{online}(1 - P_e)/(\lambda/\mu))\rho \\ &= (N_{online}(1 - P_e)/(\lambda/\mu))(\lambda/\mu N) \\ &= N_{online}(1 - P_e)/N \end{aligned}$$

APPENDIX II
NORMALIZED NETWORK THROUGHPUT

This appendix derives an expression for the normalized network throughput of an Interference Avoidance based optical CDMA LAN. First, a concise representation of line state which allows easy mathematical manipulation is defined. Using this state representation, expressions are derived for the number of codewords on the line and the probability of error when the system is in any state.

A. State representation

The state of the line can be represented by a pair (n_0, n_1) where $n_0 < N$ and $n_1 < N$. n_0 is the number of zeros in the true state and n_1 is the number of ones. The number of overlaps is n_{ov} = N - (n_1 + n_0). The term *state* will be used to refer to this reduced representation of the state of the line. The state of the line could be any value (n_0, n_1) where $0 \leq n_0 \leq N$ and $0 \leq n_1 \leq N$. A *valid state* is defined as a state where $n_0 + n_1 \leq N$. All other states are *invalid* (when $n_0 + n_1 > N$) The *initial state* is defined as the state of the line when no codewords are on the line *i.e.* $(N, 0)$. A *reachable state* is defined as a state which can be reached from the initial state by a series of state transitions due to packet arrivals. The initial state $(N, 0)$ is, by definition, a reachable state. The set of reachable states depends on the transmission scheduling algorithm. When there are no codewords on the line *i.e.* no node is transmitting packets, the state at a point on the line is the initial state *i.e.* (N, 0), *i.e.* N zeroes and 0 ones and overlaps. The arrival of a single codeword adds w '1 chips' to the state of the line and the state changes. This is called a *state transition*. Let the start state of a state transition be ($from_{n0}$, $from_{n1}$) and the destination state be (to_{n0}, to_{n1}). A state transition may be caused only by a packet arrival or departure[4]. When a codeword is added to the line, w 1 chips are added. A '1 chip' could overlap with a 0, 1 or an overlap. Let the number of '1 chips' overlapping with 0s, 1s and overlaps be c_0, c_1 and $c_{overlap}$ respectively.

[4]The effect of ON-OFF keyed modulation is neglected by assuming that a packet consists of only 1 data bits *i.e.* all data bits are ON. Packet arrivals and departures are assumed to be the sole cause of any state change. This assumption means that the probability of error calculated is higher that the true value and the throughput is the worst case throughput.

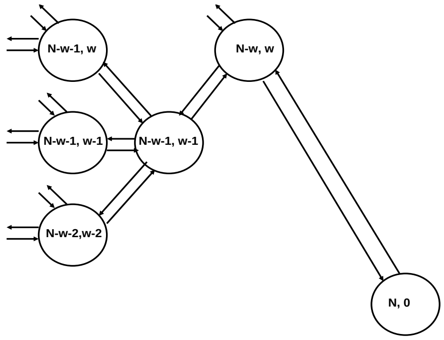

Fig. 16. State transition diagram

Then,

$$c_0 = from_{n0} - to_{n0}$$
$$c_1 = to_{noverlap} - from_{noverlap}$$
$$c_{overlap} = w - (c_0 + c_1)$$

A *valid transition* is defined as a transition where the start state and the destination state are valid, reachable states and

$$c_0 \geq 0$$
$$c_1 \geq 0$$
$$c_{overlap} \geq 0$$

An *admissible transition* is defined as a valid transition which is permitted by the transmission scheduling algorithm. A *same state transition* is defined as a transition from a state to itself. A *state transition diagram* can be drawn based on the admissible transitions. Figure 16 shows a portion of a state transition diagram for a (N, w, κ) codeset. Because of the large size of the diagram, only a few states are shown. Invalid states are not indicated in the diagram. The diagram shows the initial state $(N, 0)$ and state transitions to a few states.

B. Probability of packet error

The probability of error depends on the state. For a state $(n_0,\ n_1)$, the probability of error is the probability that for any codeword, all its '1 chips' overlap with other 1 chips or overlaps. To calculate the probability of error, the locations of the '1 chips' in the codeword are assumed to be chosen random *i.e.* the codeset construction uses $\kappa = w$. For any selfish algorithm, in state $(n_0,\ n_1)$, if $N_{online}(n_0, n_1)$ codewords are multiplexed at a point on the line, then at most n_1 codewords are transmitted without error. Therefore the probability of packet error can be approximated by the expression below:

For valid states,

$$\begin{aligned} P_e(n_0, n_1) &= 0 \qquad \text{when } N_{online}(n_0, n_1) \leq n_1 \\ &= \frac{(N_{online}(n_0, n_1) - n_1)}{N_{online}(n_0, n_1)} \qquad \text{otherwise} \end{aligned}$$

For invalid states,

$$P_e(n_0, n_1) = 0$$

Note that a packet is considered lost if there are other codewords on the line whose chips overlap with all w chips of the packet's codeword. The other codewords on the line are assumed to be ON *i.e.* transmitting 1 bits although that may not necessarily be true. Therefore the calculated packet error rate is the worst case packet error rate.

C. Number of codewords multiplexed at a point on the line

Consider a graph where the nodes of the graph are the states and each state transition due to an arrival forms a directed edge (neglect same state transitions). For any selfish transmission scheduling algorithm, this graph is a directed acyclic graph. In the graph, each edge represents the arrival of exactly one packet. Therefore the number of codewords on the line for a state $(n_0,\ n_1)$ depends on the number of edges from the initial state $(N, 0)$ to (n_0, n_1). Both the shortest and the longest path may be calculated in polynomial time. The length of the shortest and longest path from the initial state to that state are lower and upper bounds on the number of codewords on the line when the line is in that state[5].

Therefore, for valid states,

$$N_{online}(n_0, n_1) \geq ShortestPath((N, 0), (n_0, n_1))$$

$$N_{online}(n_0, n_1) \leq LongestPath((N, 0), (n_0, n_1))$$

For invalid states,

$$N_{online}(n_0, n_1) = 0$$

D. Normalized network throughput

From Appendix I, the normalized throughput in a state $(n_0,\ n_1)$ is given by

$$Th(n_0, n_1) = (N_{online}(n_0, n_1)/N)(1 - P_e(n_0, n_1))$$

where $N_{online}(n_0, n_1)$ is the number of codewords on the line at a point on the receive fiber in any state and $P_e(n_0, n_1)$ is the probability of error in any state.

The average normalized throughput at a given offered load is given by

$$Th_{norm} = \sum_{n_0=0}^{N} \sum_{n_1=0}^{N} P_{state}(n_0, n_1) Th(n_0, n_1)$$

where $P_{state}(n_0, n_1)$ is the probability of being in state (n_0, n_1) at equilibrium.

APPENDIX III
EQUILIBRIUM STATE PROBABILITIES

The equilibrium state probabilities are calculated by modeling state transitions as a Markov chain.

[5]This work assumes the number of codewords on the line is equal to the lower bound. Therefore it is a worst case assumption and the calculated throughput is a lower bound on the achievable throughput.

A. Assumptions

The analysis assumes perfect state estimation. The only reason for a state transition is an arrival or a departure of a packet. Packet arrivals are assumed to be Poisson arrivals and packet lengths are exponentially distributed. The distribution of the destination's codeword is uniform over the codeset. Under this assumption, the probability of transitioning to a particular state on an arrival is dependent only on the current state and not on the path taken to reach that state. The probability of departure to a state is assumed to be proportional to the rate of arrival from that state. Then the next state is dependent only on the current state and not on the path taken to reach that state. Under these circumstances, the state transition diagram for arrivals and departures is a Markov chain. Equilibrium probabilities may be found by solving the balance equations for the system.

B. Admissible transmissions

This subsection describes how to identify admissible state transitions given a codeset and a transmission scheduling algorithm.[6]

1) Aloha-CDMA: For Aloha-CDMA, all transitions are admissible.

2) Pure selfish scheduling: A transmission is admissible if,

$$c_0 \geq 1$$

3) Threshold scheduling: A transmission is admissible if

$$\begin{aligned} &c_0 \geq 1 \\ &to_{noverlap} \leq \alpha N \end{aligned}$$

where α is the threshold parameter.

4) Overlap section scheduling: A transmission is admissible if

$$\begin{aligned} &c_0 \geq 1 \\ &to_{noverlap} \leq to_{n1} \end{aligned}$$

C. Arrival state transition probabilities

For an admissible transition,

$$P_{arr}(from_{n0}, from_{n1}, to_{n0}, to_{n1}) = N_{admitted}/N_{total}$$

where,

$$N_{admitted} = \binom{from_{n0}}{c_0}\binom{from_{n1}}{c_1}\binom{from_{noverlap}}{c_{overlap}}$$
$$N_{total} = \binom{N}{w}$$

For a non admissible transition,

$$P_{arr}(from_{n0}, from_{n1}, to_{n0}, to_{n1}) = 0$$

D. Departure state transition probabilities

The departure probabilities from one state to another state are proportional to the arrival rates into the state. Therefore,

$$\begin{aligned} &P_{dep}(s_0, s_1, d_0, d_1) \\ &\quad = \frac{P_{arr}(d_0, d_1, s_0, s_1)}{\sum_{n_0=0}^{N}\sum_{n_1=0}^{N} P_{arr}(n_0, n_1, s_0, s_1)} \end{aligned}$$

[6] If a packet cannot be transmitted upon arrival, the system undergoes a same state transition and the packet is dropped. During a same state transition the number of packets on the line does not change.

E. Balance equations

When the system is in equilibrium, the flow into any state must equal the flow out of the state. Therefore for a valid, reachable state $(s_0,\ s_1)$,

$$\begin{aligned} &(\lambda + N_{online}(s_0, s_1)\mu)P_{state}(s_0, s_1) = \\ &\sum_{n_0=0}^{N}\sum_{n_1=0}^{N} P_{state}(n_0, n_1)\lambda P_{arr}(n_0, n_1, s_0, s_1) + \\ &\sum_{n_0=0}^{N}\sum_{n_1=0}^{N} P_{state}(n_0, n_1)N_{online}(n_0, n_1)\mu P_{dep}(n_0, n_1, s_0, s_1) \end{aligned}$$

Also,

$$\sum_{n_0=0}^{N}\sum_{n_1=0}^{N} P_{state}(n_0, n_1) = 1;$$

These equations can be solved for the equilibrium state probabilities $P_{state}(s_0, s_1)$.