Experiments in Flat Glass Cutting Optimization

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ABSTRACT:

The flat glass cutting problem is in the class of NP-Complete problems, thus making an exhaustive search, of every pane in every position, impossible. Instead, an approximate solution is formed, using a combination of software techniques, involving preprocessing of the lists to be cut as well as various feedback processes within the cutting algorithms. Due to practical considerations, the methods used are further restricted to operation on a small personal computer, using an hour or less of processing time, so previously posited solutions are not applicable. This experiment will be limited to polynomial - timed solutions, and operates on large (up to 500 panes) data sets. Current results yield a daily average efficiency of up to 89%, excellent when compared to the industry yearly average of 85%.

Experiments in Flat Glass Cutting Optimization is an investigation of algorithms for the positioning of patterns in the cutting of window panes from stock glass sizes (Fig. A). Through the testing and analysis of various cutting algorithms, insight into the general problem was obtained, and possible solutions posited.

SAMPLE PATTERN FOR CUTS OF 'A', 'B', & 'C':





These experiments began as a local glass cutting company, Mesko Glass, desired to automate their processes. They wanted to automate the positioning of the cuts, in order to minimize the waste generated in the cutting process. Most of the programs commonly available at the industrial level for such patterning requires expensive mainframe computers; Mesko, being a small firm, wanted to know if similar results could be obtained using their small IBM personal computer.

In general, the glass-cutting problem has been researched before, however much of this research proved inapplicable to Mesko's case. The classic, polynomial-based partial solutions rely on the cutting of enormously large stock sizes, where it would not be unusual to cut 50 to 500 panes from a single piece of stock glass[1,2]. Mesko, however, performs its cutting manually, and thus their pieces of cutting stock must be small enough to be easily handled by one or two glass cutters. Rarely are their stock pieces cut into more then 7 or 8 orders. Also, 'classic' solutions rely on large processing times, these expecting to cut the same set of orders over some period of time, thereby justifying the processing overhead[5,8,13]. Mesko must cut each day's orders as they are received, so their order list is substantially smaller, and varies greatly. The processing time their patterning program cannot exceed one hour or so, of including data entry, program execution for efficiency optimization, and print-outs adequate to serve as cutting guides.

In addition, Mesko has little funds for computer services, thus the program must be designed to operate on a very small computer, not a large mainframe. This severely limits the method of experimentation which is plausible. For instance, one area of interest, a few years ago, was linear programming. It requires vast amounts of memory space and processing time, and must be abandoned in this approach[9].

In order to determine the feasibility of this project, a preliminary program was written, in PASCAL on an IBM PC in January, 1984. This program experimented with two viable cutting methods, and helped demonstrate the feasibility of glass fitting on a personal computer. In addition, the amount of waste generated by this crude attempt closely approached Mesko Glass' overall efficiency, proving that a competitive solution was not far away. This project arose from the desire to increase that initial program's efficiency, through experimenting with the fitting algorithms.

Our desire for the solution to the fitting problem is not limited to this specific instance. The flat glass fitting problem is part of a larger set of computer science problems, the NP-Completes. These problems, by their nature, cannot be solved by 'brute force'. We could not, for instance, try every piece of glass in every position, to find the answer. If we did, our program might require literally years of computer time to solve, even for a relatively small list of panes. Instead, we are required to try other solutions. These may not lead to the ultimate answer, of absolute minimum waste, but they can come close enough to the answer, without requiring days of computer time, to provide a viable alternative. These are the solutions we will be attempting to discover, through our experimentation.

The algorithms utilized in the search for waste minimization fall into three major classifications. First are those methods which always tend to increase efficiency, due to the nature of glass cutting itself. Second are those heuristics which rely on information derived from the set of input data, where the operation of the algorithm adjusts to accommodate the fluctuations of the data. Finally are those algorithms which rely on feedback from the cutting process itself, to determine how the next stock will be cut.

In addition to these algorithms which constitute the actual cutting process, several techniques of pre-processing are used. One area of pre-processing, which has become an integral part our research, involves the partial ordering of the panes, in order to determine a priority in selection. Since the panes are twodimensional, simple linear sorts do not provide the proper ordering, thus other methods are being investigated.

In addition to information supplied by various coded 'test' procedures, such as a scatter-plot, interviews were conducted of Mesko's more experienced glass-cutters, to learn how they approach the problem during their work.

The experiment was performed under a list of presumptions, derived largely from interviews with professional glass-cutters. These rules include:

- 1. Always try to fit the largest piece in the list to be cut to the remaining plates of glass. If it doesn't fit, proceed to the next largest size.
- 2. Start with the smallest piece of glass available, and use larger stock sizes only when necessary. Although this will reduce efficiency, it is cost effective, since smaller stock sizes are much easier to handle, and thus less likely to shatter in transport.
- 3. Always place cuts orthogonally, to the outside of the plate (known as 'normalization'). While this may lead to inefficient patterns[5], it will be assumed for program simplicity.

The exploratory algorithm, designed to test the feasibility of such a cutting algorithm on a personal computer, operates on three assumptions. First, the largest order must be cut at some time; if it is cut first, there is more of a chance of finding other pieces which will use the scrap generated from that initial cut. Second, the scrap generated should be filled by the largest piece immediately, rather than fill it with two smaller pieces, which may or may not exist. Third, when cutting a piece of glass from the stock, cut it in such a way as to leave the two most usable scrap pieces (denoted as cut_type A - see Appendix A). This method uses the following algorithm:

PRELIMINARY CUTTING ALGORITHM

- I. SORT the stock list into a list of small to large.
- II. SORT the orders list into a list of large to small. (note that the sorting was performed by the naive technique of sorting by larger side first, then by smaller side within the larger.)
- III. LOOP:
 - A. get the largest order from the top of the order list.
 - B. get the smallest stock which can be used to cut that order from.
 - C. send the stock thus chosen to the recursive cutting procedure:
 - cut the largest order (first one found by a linear search of the list) which fits inside that piece of stock, using the method denoted as 'A'.
 - 2. remove the order thus cut from the list of orders to be cut.
 - 3. set up the cut tree to reflect the cut thus made.
 - recursively call to 1, using the remainders of stock generated by the cut as new stock sizes.

If the stock cannot be cut (the list is searched, and no fit is found) return out of the recursion.

For these tests, sample stock sizes and orders were obtained, from one month's business of Mesko. Using these orders and stock sets resulted in 79.8% efficiency. In initial investigation of the output generated by this method, an overuse of the smaller stock sizes was noticed. The efficiency seemed to be lower for these stock sizes. The initial reaction was to remove the smallest stock size from the stock list, and to re-run the program, to see if there would be any noticeable difference.

Using preliminary orders set, and the reduced stock set, resulted in 83.2% efficiency (the stock list was revised to exclude the smallest stock size, stock #1). The observations of this trial was a continued overuse of the smaller stock sizes, even though the efficiency did increase slightly. The conclusion was to remove both stock sizes 1 and 2, the smallest two sizes of the original data set. Using preliminary orders set, and newly revised stock set, resulted in 80.7% efficiency (the stock list was revised to exclude the smallest two stock sizes, #1 and #2). The observations of this trial was a continued overuse of the smaller stock sizes, and a note that the efficiency now dropped slightly. The conclusion was that removing the smaller stock sizes was not a useful option. There must be a reason the industry uses the smaller stock sizes, other than mere handling requirements. Small stock sizes clearly must be used, but discriminately. The question is how to decide when to use them.

Reevaluating data obtained from the three above experiments, a rough graph was hand sketched of the efficiency of each piece being cut (Fig. B). A visual evaluation of the graph thus sketched showed that some of the stock sizes have 'erroneous' efficiencies.

The efficiency drops, in a generally decreasing fashion, for each stock being cut. Most of the stock being cut does not err from this steady, slightly decreasing rate.

In the first case, using the original test data, 6 of the clearly erring pieces of stock have efficiencies less than 75%. Of these, all use stock size #1. This explains the increased efficiency seen in trial 2, where stock #1 was removed from the list.

In the second case, only 3 of the stock pieces have efficiencies less than 75%, and of these both are of stock #2. This explains why removing stock #2 as well as #1 does not increase the efficiency. Not that many of the cuts executed were affected by removing stock #2.

A major concern at this point was to obtain a higher efficiency using the smaller stock sizes, to justify their use by the industry.

A new strategy was created: to recut the orders from a larger stock size, if the efficiency was 'out of line'. Temporarily, 'out of line' will be defined as having an efficiency less than 80%. This method was denoted the 'backup' method, since, if the efficiency was errant, the cut would be 'undone', or backed up, and then recut using a larger stock piece.

Preliminary analysis proved the viability of the 'backup' method. However, more than one data set must be analyzed in order to determine whether the method is viable or not.

This analysis shows that a backup on efficiencies less than 80% yields an overall efficiency of 85.3%, which is excellent, compared to reported industry standards of 85% (yearly) efficiency[13].

When in the process of backing up, if selection of the stock goes 'off the end of the list', it is best to choose the first stock in the list. This rule is based on the statistical evidence that the first stock chosen is usually the best in the list. Even if that piece is not the most efficient, it is the smallest. Since this set of cuts will generate an unacceptable level of waste anyway, it is wisest to misuse the smallest, to reduce the effect on the overall efficiency (Table C).

In order to properly test the program and how it functions, a variety of data must be utilized. In the initiation of this project, Mesko Glass Company was contacted, as a possible user of Figure B



Table C

Number of panes	Effic. %	# "off end"	New Effic.	# first is best
60	71.9	7	79.9	6
100	82.8	8	84.9	7
150	88.5	3	89.0	0
200	87.4	1	87.5	1

the final product. In exchange for the final program, Mesko was to aid us in the development of the user interface of the program, and to provide us with test data, to be used in testing the functioning of the program. However, due to administrative difficulties, data from Mesko was not available in the quantities necessary to properly test the algorithms.

This development has hindered our initial experimental design plan. We had hoped to use actual industry data to evaluate our code. However, since our principle source of data was eliminated, it became necessary to obtain data from an alternate source.

In order to generate the volumnuous amounts of data needed to properly test the program, it was decided to generate the data files using a random number generator. However, in order to do such, a generator had to be located and tested as being truly random.

The focus of the project was diverted to the evaluation of the available random number generators, in order to determine which would be best for our needs. Three generators were at our disposal: the MODULA MathLib0 generator (included in the runtime support package), a generator translated from a TI-58C handheld calculator, and one written by Dr. J. Beidler.

Three tests are traditionally performed on random number generators, to determine the randomness of the numbers they generated. The three tests are the Frequency test, the Serial test, and the Gap test. The Frequency test counts the occurrences of each number generated, where an even distribution shows frequency independence. The Serial test plots two successive random numbers, in (x,y) fashion. It is used to find sequential predictability in the generator. The Gap test measures the gap between a number's occurrence, and its last previous occurrence. It is designed to test the cyclic tendencies of the generator.

The TI-58C calculator generator failed all three tests; it showed definite cycles, did not generate an even distribution, and was accurately predictable. The MODULA MathLib0 generator passed all three tests successfully. It showed true randomness, and was thus suited to our needs. Dr. Beidler's generator was not tested, since a standard MODULA facility could satisfy our requirements.

It is useful to note that we may not desire a truly random or evenly random generation of panes to be cut. We may want skewed or 'normal' distributions for our tests. However, if we have a true random generator at our disposal, we can create normal or skewed distributions of any weighting.

Some questions to be addressed, when randomly generating the data files are:

- A. Should the cut list be evenly distributed, or skewed? If skewed, how so?
- B. Should the sizes generated be thoroughly random, are merely random selection of standard sizes? What are the 'standard' sizes?

C. Should the stock be generated:

- 1. simultaneously with the orders.
- 2. independently from the orders.
- 3. kept constant.

If the stock is generated, how should it be distributed and selected? (see A,B).

Within the program, there are provisions for plotting a cluster graph, of height/width ratio vs. area (see Appendix C). It may be beneficial for the program to recognize clustering in certain areas of the graph, such as pieces with large height-to-width ratios and large areas, so that these pieces may receive special consideration when deciding which cutting method (A,B,C,D - see Appendix A), or which order of selecting the cuts should be used. The program should be equipped to handle all cases of clustering, if and when it occurs.

A parallel issue to cluster programming is the generation of data to properly test the clustering program. These considerations further complicate the generation of new data sets.

A program has been developed to generate evenly distributed order data sets. For the initial testing, the program will generate panes whose dimensions lie between 12 inches and a maximum dimension specified by the user.

The preliminary stock data will be used as the stock data set for these initial testing purposes. A full listing of this stock set appears in Appendix B.

The number of panes generated in the cut orders file will be specified by the user. Also specified by the user is the seed integer to be used in the generation of the random numbers, facilitating user control of the generation sequence. Since the seed can be recorded, data sets can be regenerated; they need not be saved in whole.

The initial data sets used to test the performance of the program have dimensions between 12 inches and 130 inches for the longer side, and between 12 inches and 90 inches for the shorter side. Thus the pieces generated will lie between one foot square and the dimensions of the largest stock size.

In partial response to the correction of a coding bug, a warning message was incorporated to alert the operator if, in the course of finding a stock whose efficiency was over 80%, the program ran out of stock sizes to 'back up' to.

It should be noted that, in the decision making process, certain decisions cannot be correctly assumed. For example, in the case of deciding the method of sorting, how can one decide whether a piece is larger or smaller? Since there are two dimensions, are they to be sorted by larger side first?

While this may seem to be a logical method of sorting, consider that we sort the panes for a purpose: to locate those panes which are largest. We wish to cut the largest, first. However, we cannot make the assumption that the longest side implies the largest. What about largest area? What do we really mean or intend by 'largest'? Similarly, we backup and recut those pieces whose efficiencies are 'out of line'. What do we mean by 'out of line'? What percent of error in this judgement can we ignore as trivial? For the final series of experiments, a total of four alternate methods of sorting the initial list of orders were devised. The original method involved sorting by longest side first, and, in the event of a tie, to sort by the smaller side. This method, called the 'long sort,' ignores any consideration of area as a criterion for sorting.

Since the efficiency in calculated by area, it seems evident that there may be merit in cutting those pieces which affect the efficiency the most, first. The method of accomplishing this is to sort solely by area, the 'area' sort.

The third sorting method, the 'four-sort', combines the consideration of cutting the largest areas first, with the desire to cut the oblong, or more 'odd' shapes first. Consider a graph, of shorter/longer dimension vs. area, relative to the area of the largest piece (Fig. D).



Figure D

Those pieces with large areas will be cut first (1),(2), but within that group, group (1), with short side/long side ratios less than 0.5, should be cut first. They are at least twice as long as wide, and as such are oblong. Next cut the large, 'square' pieces, (2). Then the small, oblong (3), and finally the square, small pieces (4). Within each group, an arbitrary 'long' sort is performed, to provide some ordering, although this is probably not necessary, due to the nature of the grouping. The four groups are then connected together, end to end, resulting in a ordering of (1)-(2)-(3)-(4).

Finally, a more continuous form of the discrete 'four-sort', the 'coefficient sort', was developed, by the formula:

[(short_side / long_side) + (1 - area / max_area)] / 2

Where max_area is the largest area of a pane, and the result is

an average of a piece's 'largeness ratio' and its 'oblong-ness.' Since the pieces are to be sorted from low coefficient to high, the area ratio had to be inverted as (1 - area_ratio).

Given the initial algorithm, two places of sorting were possible. The 'first sort' determines the order the panes are in when the decision is made to 'cut the largest piece', which becomes the top of that list. The second ordering is in the 'filling' algorithm, where the stock and its subsequent scrap is utilized. This is when the decision is made to search the list 'to find the largest order which fits the remaining scrap.'

For the final testing, the decision to 'backup' on inefficient patterning was also optional, resulting in the following experimental design:

first	-			fill				backup
long	\backslash		/	long	\setminus			
coef	\setminus	х	/	coef	\backslash	х	/	yes
four	/		\setminus	four	/		\setminus	no
area	/		\backslash	area	/			

These methods were tested in the original algorithm, which can essentially be described as a 'first fit decreasing' scheme. Each test run was performed on a fixed set of randomly generated orders, with lists ranging from 50 orders to 200 orders, in increments of 10. Each of these sizes was run with 10 data sets, the results being averaged. Thus a full test as this point involved:

- 4 first sorts * 4 fill sorts * 2 (backup/no backup) = 32 program types
- 16 data set sizes * 10 data sets per size = 160 complete data sets (50..200)

32 program types * 160 complete data sets = 5120 full program runs

5120 program runs * 30 seconds estimated average CPU time per run = (VAX 11/780, running MODULA2)

42 hours, 40 minutes of CPU time

Repeated program runs were not feasible, since our school has a single VAX to serve over 400 Computer Science majors, various other users, such as accounting classes and word processing courses, and faculty. In fact, much of the time spent involved circumnavigating the system constraints designed to prevent any program from using as much CPU time and space (25 Megabytes-to store raw data and output) as was necessary for only *one* of these test runs.

The data from these experiments were tabulated and several averages reported, one average for each data set size. The two most significant figures were average CPU time, and average patterning efficiency. In addition, two other values which measure the algorithm's effectiveness were the average number of 'backups' performed while cutting the panes, and the number of times the backing up resulted in 'going off the end' of the list of stock sizes. This data appears in Appendix D.

In order to effectively evaluate the volumnuous data from these trials, an algorithm was designed which correlated various values, using the method of least squares. In order to determine which type of function best approximates the relationship of the two correlated variables, not only were the actual values correlated, but their logarithms, inverses, etc. The following were determined to provide an adequate sampling of the probable relationships the variables could contain:

Correlate		obtaining slope (m) and intercept (b).	Giving a relationship		
X VS	з Ү	which then represent:	of:		
Х	Y	Y = m * X + b	linear		
Х	log(Y)	Y = e^mX * e^b	exponential		
log(X)	Y	$Y = log(b * X^m)$	logarithmic		
log(X)	log(Y)	Y = b * X^m	polynomial		
log(X)	1/Y	Y = 1 / log(b * X^m)	inverse logarithmic		
1/X	log(Y)	Y = e^(m / X) * e^b	inverse exponential		
X*log(X)) Ү	Y = m * X*log(X) + b	nlog(n)		
Х	Y*log(Y)	Y*log(Y) = m * X + b	(added for symmetry)		

Each set of data pairs to be evaluated were correlated by these eight methods. The method with the highest correlation coefficient (closest to 1 or -1) best expresses the relationship between the coordinates. Note, however, that this determines only the major relationship between the pairs of data. The data may be related in multiple ways, such as both linear and exponential, i.e.:

$$Y = 3 * X + 0.1 * X^{4.1}$$

In this case, the linear portion of the curve is dominant for small values of X, while at larger values the exponential nature of the expression is evident. For these purposes, finding the dominant relationship is sufficient.

It was predicted, and experimentally proven, that the best correlations would be data set size (number of panes) vs. efficiency, and data set size vs. CPU time. The efficiency follows an inverse exponential with respect to set size, and the CPU time follows a polynomial relationship to set size.

Efficiency is an inverse exponential for obvious reasons.

As the data set size increases, the chances of finding orders to be cut which will utilize the scrap generated by other cuts increases, until a maximum is reached. The maximum here is expressed by the constant e^m.

The CPU time is a polynomial by program design. The desire was to find a polynomial based algorithm for relatively efficient patterning, by testing various polynomial-timed algorithms. The highest power of the polynomial is 'm'.

A condensed listing of these factors (of polynomial time constant, and efficiency asymptote), appears in Table E. Program method is listed as first sorting method, fill sorting method, where L = long sort, C = coefficient sort, F = four sort, and A = area sort. A full listing of the correlations appears in Appendix E.

Table	Е

		With	bac	ckup		Without	backup	DEI	TAS
Program		CPU		effic.		CPU	effic.	CPU	effic.
Method		poly		limit		poly	limit	poly	limit
L L	1	2.56	*	90.6	17	2.77	89.1	.21	-1.5
СL	2	1.78		77.6	18	2.08	77.9	.30	-0.3
FL	3	2.30		85.6	19	2.51	83.9	.21	-1.7
A L	4	1.93	*	90.3	20	2.15	88.8	.85	-2.5
- a	-	1 00			0.1	0 1 1			2 0
ΓС	5	1./3		79.2	21	2.1/	76.0	.44	-3.2
СС	6	2.20		84.4	22	2.69	76.4	.49	-8.0
FC	7	2.00		83.1	23	2.42	77.0	.42	-6.1
A C	8	1.69		84.3	24	2.13	75.4	.44	-8.9
т. F	۵	2 31		85 1	25	2 53	77 9	22	-7 2
	0	2.51		71 7	25	2.35	69.8	.22	_1 9
	1	2.12		/ I . /	20	2.35	09.0	. 2 3	-4.9
		2.23		03.5	27	2.04	77.7	.41	-5.0
AF' I	2	2.16		84.8	28	2.42	/9.6	.26	-5.2
LA 1	3	1.95	*	90.4	29	2.14	90.3	.19	-0.1
C A 1	4	1.88		78.7	30	2.12	78.7	.24	0.0
τ Δ 1	5	2 14		89 1	31	2 28	86 6	14	-2 5
	6	2.11	*	01 0	33	2.20	99.0	20	_2.5
A A I	0	2.00			22	2.00	00.T	. 40	4.0

This table gives some very interesting results. First, all of the efficiency limits dropped when the backing up process was removed, giving more than a single instance of improvement. Backing up seems, in every case, to be at least somewhat beneficial. Second, the four highest efficiencies (*) are for the area and long sorts, in various combinations, thus proving the superiority of these sorts, for this algorithm only. Other algorithms may give different results, so this would not be a reason for abandoning the other sorting techniques.

In the area of timing analysis, several startling results were obtained. First, all of the timings proved polynomial, as expected, but of orders between 1 and 3, whereas the predicted limit would be 2 (a squared polynomial relationship). The lower bound of 1 is expected. In addition, the backup algorithm was consistently 'squarer', while the non-backup algorithm behaved more as cubed. This implies that the true timing has both squared and cubed terms, and the backing up process adds another squared term, thus increasing the squared term's influence over the behavior of the polynomial.

This is consistent with the timing analysis (Appendix A). The algorithm itself has cubed timing, while the sorts all have squared timing (worst case). Since the backup method executes more sorts, every time it 'backs up,' it should be more influenced by the squared term of sorting, and thus show a 'squarer' curve. The non-backing up algorithm did not execute as many sorts, and would thus have a smaller squared term, permitting the cubed term to dominate the curve.

Note that while the polynomial was of a higher degree without backing up, backing up did increase the CPU time required to run the algorithm, in every case, as would be expected.

The first variations of this experimental design involved altering the set of stock sizes, from which all panes are cut. One variation, stock set 'Two', involved halving the largest piece of the original stock set, until it was too small to be usable. Another variation of the stock data set was a totally randomly generated set. Since the previous experimental design required over 40 hours of CPU time to run, the design was restricted to the starred (*) sorting methods which proved most viable above. The condensed results of these two tests appear in Tables F and G, while the fully correlated results appear in Appendix E. A listing of the stock data sets, in full, appears in Appendix B.

Table F: Second, Created stock set

Program Method	With bac CPU poly	ckup effic. limit	Without] CPU poly	oackup effic. limit	DEI CPU poly	JTAS effic. limit
LL 1 AT. 4	2.47 1 98	88.0 17 87.6 20	2.67	 86.2 87 5	.20	-1.8
LA 13	2.03	87.9 29	2.14	88.1 87 F	.11	+0.2

Table G: Third, Random stock set

		With	backup		Without	backup	DEI	LTAS
Program	n	CPU	effic.		CPU	effic.	CPU	effic.
Method		poly	limit		poly	limit	poly	limit
 L L	1	2.49	88.2	17	2.72	85.2	.23	-3.0
A L	4	1.99	87.5	20	2.13	86.4	.14	-1.1
LΑ	13	2.02	90.0	29	2.17	87.3	.15	-2.7
ΑA	16	2.59	88.3	32	2.72	86.0	.13	-2.3

Note how both of these trials generate similar results, with efficiencies maximized at about 90 %, or 10 % waste. It seems that, within reasonable limits, the contents to the stock set is largely inconsequential.

For all of these trials, the stock set had to first be sorted, just as did the orders set. The algorithm performed only the long sort on the stock list. While it may seem presumptive not to vary the stock sorting as well, it was determined early in the experiment, by coincidence, that all four of the sorting techniques yield identical orderings. This was true for the first stock set only, but since the number of stock pieces was small, the ordering was deemed inconsequential, provided there was a general trend of small to large.

It is also useful to note the accuracy of these figures. In addition to the averages computed, standard deviations, of N-1 weighting were also calculated. Representative values of these SD's appear in Table H.

Table H

Data Item:	Range of SD:
EIIIClency	0.8 % - 3.0%
CPU time	2 % - 15 %
Backups	50 % - 1000%
Off end	50 % - 200 %

From these figures, we can assume that the efficiencies are relatively stable, for a constant orders set size. The CPU time similarly maintains a fairly tight average, thus these two averages can be used with some degree of confidence, although it should be remembered that they are averages, not actual data.

The number of backups seemed to remain unpredictable; it was not related specifically to the number of orders processed, nor was it related to the efficiency. In fact, within the set of data from one sorting method, the number of backups seemed relatively constant.

The number of times the search for a more efficient stock size went 'off the end' of the list of available stock was similarly unpredictable. It fluctuated wildly, even within one sorting method; in fact, some of the SD's were as much as twice the value of the average. This number will exceed the number of backups on occasion, contrary to common sense. This is because it is common to begin the search for a stock size with the last item in the stock list. When this size fails to yield an acceptable efficiency, the backup immediately fails 'off the end' of the list. Due to the current implementation, this increments the 'off end' count, but not the 'backup' count.

The resultant graphs appear in Appendices F & G. Notice in these graphs how, visually, the efficiency seems to approach a limit (Appendix G), and how the CPU time graphs behave, some rising up more quickly than others. The graphs may be overlaid for comparison.

Some other methods suggested for experimentation, both by the results generated here, and by the currently available literature (see bibliography) include:

- 1. Selectivity in choosing cut method. The present algorithm utilizes only one of the four possible orientations. Other orientations may be more efficient, if the selection process were reconfigured.
- 2. A thorough research of the variations in the generation of the stock set, studying not only what combinations yield the best efficiencies, but also how the industry cost of the different sizes (price per square foot) would affect the selection of stock sizes. While overall waste should be reduced, a less efficient (area wise) pattern may be more cost effective to the company.
- 3. A more thorough testing of the algorithms, using more data sets. Averages presented here were of ten data sets; this is not as statistically significant as would be desired.
- 4. Testing of the algorithms in an industrial setting, with actual data sets and stock sets as used daily by a company in operation. This kind of field testing is the only true test of an algorithm's validity.
- 5. Break the process into two 1-D problems. The result is commonly known as the 'strip-method'.
- 6. A fine tuning of the methods presented here, via searching all stock sizes, and choosing the most efficient at each decision, rather than 'backing up', for instance.

This experiment, while not as conclusive as desired initially, has shown very promising exploratory results. Several sorting methods have been investigated, some ruled inefficient, others surprisingly viable (especially the long and area sorts). The location of the sorting doesn't seem to affect the performance of the algorithm significantly; subsequent experiments may abandon this approach.

Other information has been obtained in the area of optimal stock sizes. While smaller, pre-cut stock sizes are considered generally wasteful, their use is necessary for handling ease by the glass cutters, and have proven to be critical in waste minimization.

The main accomplishment of this experiment has been to show that some very simple heuristics can yield industry-rivaling results. This is also the first time the issue of polynomial timing of the cutting algorithm has been given first consideration when designing a solution, resulting in an algorithm which can easily handle both small and large numbers of orders, in very small times. This algorithm, with all its testing procedures and monitors built in, runs 20 times faster than similar algorithms, on similar data sets[8].

I would like to thank the University Library for locating the necessary research materials, Mr. Parker, of the Computer Center for allocating the required space and privileges to run the program, and Dr. Beidler for his guidance and direction. Finally, I would like to thank the students of the University, for their input and evaluations of this project, and for their patience during the many times these programs brought the system down.

APPENDICES

- A Timing analysis and proof of NP nature
- B stock data sets used
- C sample clustergraphs and pattern outputs
- D raw statistical data
- E data correlations
- F CPU time vs. number of panes graphs
- G efficiency vs. number of panes graphs

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APPENDIX A: Timing analysis and proof of NP nature

The glass-cutting problem is a special case of twodimensional bin-packing, an NP-complete problem. Glass cutting adds the restriction of guillotine-cutting, which implies that all cuts to a piece of stock must sever the stock in two; a cut cannot stop in the middle of a piece of glass. Also, the glass cannot be cut into strips narrower than 1/2". Finally, there are several sizes of stock to be used, rather than a single 'bin' size.

Given the restriction of guillotine cutting, there are, at most, four unique ways to position each piece of glass to be cut onto the stock (see Fig. A).

The proof of NP is as follows. At the start, there are N possible orders to be cut from M unique stock sizes. One of the N panes is cut, from one of the M stock sizes. It is cut in one of the four possible orientations, yielding two additional sizes of stock, as scrap to be reused. Now N-1 pieces remain, to be cut from M+2 sizes of stock glass. The process repeats, for N levels, until one of the infinite processes remaining contains the path to the solution with minimal waste. The number of levels is thus 4N, to find the solution (Fig. B). Since this process is performed non-deterministically, we do not know which of these branches to perform first; we are assuming to perform all of them in parallel. On a serial machine, this would imply at least an exponential timing, of 4^N. One of these branches contains the answer; the problem lies in deciding which route to take, without creating the entire solution tree[5].

Since there are N levels to the infinitely parallel solution of this problem, given non-deterministic processing, the problem is said to be non-deterministically polynomial.

Figure A

THE FOUR WAYS TO CUT A PANE FROM STOCK GLASS





Figure B

(to the Last level, where solution can be determined.)

In the analysis of the algorithm's timing, there are two areas of consideration, the main cutting loop (recursive), and the various sorting procedures.

Each of the sorts is a variation on the Quicksort, with increased timing on the order of N. The sorts thus have a timing of the Quicksort, or NlogN to N^2 .

The algorithm has timing of order N^3. This can be shown by an analysis of the algorithm:

1 Loop N times, to find the 'largest' pane to be cut. 2 Loop N times, in executing the recursion. 3 Loop N times to find a pane to fill the stock, at that level of recursion. END END Three loops of N, nested thus, result in N^3 timing. The two loops which locate panes (1,3) obviously have timing of order N, since they are searches of linear lists. Not so obvious, however, is that the recursion, noted here by 'loop' 2, is of order N. It has been observed that, as the number of panes increases, the chance of filling the potential waste area on a pattern increases, since there is more of a chance of a piece small enough to be utilized, existing. While this does not imply that the recursion is related to N by design, it does have an order N due to effects of probability. The larger N is, the deeper the recursion is likely to go.

Thus the entire algorithm has a timing of order N^3 , with the sorting of order N^2 . This gives rise to some interesting results, especially when the number of sorts is increased by backing up.

Whenever a cut is 'backed up,' the cut is disassembled and sorted back into the available panes list. This drastically increases the number of sorts performed during one program run. The timing of the algorithm then begins to exhibit an N^2 timing, since the N^2 factor is increased, while the N^3 factor remains unchanged.

APPENDIX B: stock data sets used

DATA SET ONE

1.	48.0	128.0
2.	60.0	128.0
3.	72.0	128.0
4.	80.0	128.0
5.	84.0	128.0
6.	88.0	128.0
7.	96.0	130.0

DATA SET TWO

1.	96.0	130.0
2.	96.0	65.0
3.	48.0	65.0
4.	50.0	32.5
5.	25.0	32.5

DATA SET THREE

1.	96.0	130.0
2.	87.2	84.1
3.	49.4	45.5
4.	70.7	73.8
5.	58.1	98.8

APPENDIX C: sample cluster-graph and pattern outputs





X axis denotes long/short ratio, Y axis denotes area





SAMPLE PATTERN B

APPENDIX D: raw statistical data

(1) first by longest side, fill by longest side, with backup

num. panes	effic.	CPU time	back ups	off end
50.0 60.0 70.0 80.0 90.0 100.0 110.0 120.0 130.0 140.0 150.0 160.0 170.0 180.0 190.0	85.1 85.9 86.1 87.2 88.1 86.4 88.9 88.1 88.5 88.5 88.5 88.3 88.7 89.1 89.0 89.3	1.79 2.58 3.62 4.28 6.35 8.74 11.1 14.2 17.0 21.0 25.1 29.8 35.7 41.3 49.6	$\begin{array}{c} 20.3\\ 25.3\\ 24.5\\ 18.3\\ 27.7\\ 38.6\\ 23.6\\ 30.6\\ 30.8\\ 33.0\\ 32.3\\ 30.4\\ 32.6\\ 25.5\\ 33.0\\ \end{array}$	2.00 3.00 2.50 2.20 5.30 0.90 3.20 3.10 2.80 3.30 2.80 3.10 1.70 3.40
200.0	89.I	58.6	34.5	2.60

(2) first by coefficient, fill by longest side, with backup

num. panes	effic.	CPU time	back ups	off end
50.0 60.0 70.0 80.0 90.0 100.0 110.0 120.0 130.0 140.0 150.0 160.0 170.0	69.7 70.3 69.3 70.6 71.4 72.8 73.3 72.7 75.0 73.3 73.8 74.5 74.8	1.44 3.00 2.87 3.99 4.28 5.41 6.78 9.68 8.80 9.23 11.8 12.9 13.7	6.90 30.8 10.8 16.4 2.30 8.30 14.7 38.2 14.6 1.10 11.0 11.8 2.90	12.2 15.3 19.4 17.8 21.0 21.8 25.4 28.3 27.7 31.1 32.2 31.8 33.3
180.0	75.4	15.7	2.90	35.0
190.0	76.0	18.1	13.0	35.4
200.0	76.0	22.3	31.4	38.4

(17) first by longest side, fill by longest side, NO backup

num. panes	effic.	CPU time
50.0 60.0 70.0 80.0 90.0 100.0 110.0 120.0 130.0 140.0 150.0 160.0 170.0 180.0 190.0 200.0	80.6 81.7 83.3 83.9 83.2 84.0 85.0 85.2 86.9 85.5 85.6 85.6 85.8 86.5 87.1 87.3 86.7	$\begin{array}{c} 1.17\\ 1.78\\ 2.77\\ 3.68\\ 5.26\\ 6.84\\ 9.33\\ 11.9\\ 14.8\\ 18.5\\ 22.7\\ 26.9\\ 32.3\\ 37.6\\ 44.3\\ 50.9 \end{array}$

(18) first by coefficient, fill by longest side, NO backup

num. panes	effic.	CPU time
50.0 60.0 70.0 80.0 90.0 100.0 110.0 120.0 130.0 140.0 150.0 160.0 170.0	68.0 67.5 68.4 69.8 70.7 71.5 72.8 70.9 74.3 73.4 73.0 74.0 74.8	0.861 1.41 1.79 2.40 2.98 3.60 4.51 6.10 6.30 7.31 9.05 10.1 11.2
180.0	74.9	12.8
190.0	75.7	14.7
200.0	75.5	16.7

(3)	first by four-sort,
		fill by longest side,
		with backup

num. panes	effic.	CPU time	back ups	off end
50.0 60.0 70.0 80.0 90.0 100.0 110.0 120.0 130.0 140.0 150.0 160.0 170.0 180.0	79.7 78.5 81.3 83.1 82.6 79.6 83.3 82.7 82.5 82.0 83.6 84.3 84.0 83.7 82.2	1.74 2.71 3.68 4.40 6.22 7.33 10.6 13.2 15.5 15.9 20.3 24.5 26.0 31.8 29.7	17.9 28.1 31.6 25.5 34.8 26.5 45.3 48.5 47.6 23.3 40.4 46.3 31.0 56.2 57.2	5.90 8.30 7.70 6.20 8.30 13.5 8.60 12.1 14.9 16.6 13.5 13.6 13.7 16.4
200.0	84.7	42.4	43.3	15.6

(4) first by area, fill by longest side, with backup

num. panes	effic.	CPU time	back ups	off end
50.0 60.0 70.0 80.0 90.0 100.0 110.0 120.0 130.0 140.0 150.0 160.0 170.0 180.0	80.8 83.8 86.2 86.3 86.9 84.9 87.3 88.2 86.4 86.1 86.5 87.4 88.4 88.5	$ \begin{array}{c} 1.50\\ 2.52\\ 3.07\\ 3.72\\ 5.44\\ 6.05\\ 7.92\\ 9.60\\ 9.64\\ 10.4\\ 12.5\\ 14.9\\ 16.4\\ 18.9\end{array} $	$\begin{array}{c} 16.1 \\ 35.3 \\ 31.3 \\ 31.0 \\ 44.7 \\ 39.0 \\ 43.7 \\ 52.7 \\ 37.7 \\ 36.8 \\ 35.1 \\ 47.1 \\ 41.0 \\ 47.9 \end{array}$	5.10 4.70 3.00 2.70 3.30 7.80 4.00 3.30 7.90 8.30 7.20 5.70 3.60 4.80
190.0	88.3	22.6	51.4	7.10
200.0	87.9	24.8	6T.T	1.90

(19) first by four-sort, fill by longest side, NO backup

$\begin{array}{cccccccccccccccccccccccccccccccccccc$	num. panes	effic.	CPU time
200.0 81.9 35.6	50.0	76.0	1.09
	60.0	76.9	1.65
	70.0	78.7	2.33
	80.0	80.0	3.20
	90.0	80.4	4.40
	100.0	77.7	5.59
	110.0	80.4	7.81
	120.0	80.8	9.38
	130.0	80.0	11.4
	140.0	80.9	12.7
	150.0	81.6	16.2
	160.0	81.9	19.6
	170.0	82.7	22.2
	180.0	81.3	25.1
	190.0	81.2	30.1
	200.0	81.9	35.6

(20) first by area, fill by longest side, NO backup

	CPU
effic.	time
78.5	1.03
81.1	1.50
82.7	2.23
84.2	2.76
83.5	3.75
82.0	4.40
84.5	5.81
85.0	7.48
85.0	7.93
85.1	8.67
85.0	10.8
85.4	12.5
86.2	14.3
86.3	16.1
86.4	19.0
86.2	20.2
	effic. 78.5 81.1 82.7 84.2 83.5 82.0 84.5 85.0 85.0 85.0 85.1 85.0 85.4 86.2 86.3 86.4 86.2

(5) first by longest side, fill by coefficient, with backup

num. panes	effic.	CPU time	back ups	off end
50.0	78.1	4.20	71.9	8.80
60.0	74.4	6.54	93.3	14.7
70.0	76.9	9.35	113.0	16.0
80.0	78.0	10.3	112.0	14.6
90.0	78.0	13.2	122.0	16.6
100.0	77.0	14.9	124.0	21.2
110.0	77.0	18.2	128.0	21.6
120.0	77.7	20.3	148.0	25.6
130.0	79.7	23.1	149.0	22.3
140.0	78.9	25.8	161.0	25.8
150.0	78.5	29.0	156.0	25.4
160.0	78.9	35.4	180.0	27.1
170.0	78.7	38.5	188.0	31.3
180.0	77.8	40.9	201.0	34.6
190.0	78.0	48.8	230.0	36.4
200.0	78.1	53.3	208.0	36.8

(6) first by coefficient, fill by coefficient, with backup

num. panes	effic.	CPU time	back ups	off end
50.0 60.0 70.0 80.0 90.0 100.0 110.0 120.0 130.0 140.0 150.0 160.0 170.0 180.0 190.0	74.3 74.2 75.5 76.3 77.4 77.9 79.1 79.3 80.3 80.4 80.5 80.8 81.0 81.0 81.8 91 5	3.74 6.12 8.52 9.51 13.7 16.6 21.8 24.2 28.0 34.7 40.6 50.2 57.4 64.9 68.0 87.2	70.0 95.2 101.0 103.0 119.0 124.0 133.0 140.0 138.0 149.0 157.0 176.0 177.0 184.0 186.0 210.0	11.7 16.4 16.3 17.7 19.3 19.1 22.2 21.4 21.9 22.8 24.6 26.3 26.9 25.8 20.2
200.0	01.0	07.2	210.0	50.2

(21) first by longest side, fill by coefficient, NO backup

num.		CPU
panes	effic.	time
50.0	70.5	1.19
60.0	69.3	1.69
70.0	71.8	2.42
80.0	70.9	3.16
90.0	71.8	4.10
100.0	72.3	4.80
110.0	72.7	6.49
120.0	73.0	7.94
130.0	75.0	9.11
140.0	73.9	10.8
150.0	73.6	12.4
160.0	74.1	14.3
170.0	73.9	16.6
180.0	73.8	18.2
190.0	74.7	21.4
200.0	73.8	24.0

(22) first by coefficient, fill by coefficient, NO backup

num.		CPU
panes	effic.	time
50.0	68.3	1.11
60.0	67.6	1.73
70.0	68.6	2.64
80.0	68.4	3.65
90.0	69.1	5.17
100.0	70.8	6.66
110.0	71.0	8.78
120.0	72.2	10.7
130.0	73.1	13.5
140.0	72.2	17.2
150.0	73.5	20.2
160.0	73.3	24.1
170.0	73.2	29.1
180.0	73.6	33.2
190.0	75.0	38.9
200.0	74.1	45.4

(7)	first by four-sort,
		fill by coefficient,
		with backup

num. panes	effic.	CPU time	back ups	off end
50.0 60.0 70.0	77.6 76.6 77.0	3.72 5.75	65.8 79.8	8.40 10.7
80.0	78.2	9.62	111.0	13.6
90.0	79.6	12.7	120.0	13.7
100.0 110.0	79.4	16.9 21.5	141.0 155.0	17.2 18.8
120.0	79.9	23.5	163.0	20.8
130.0	81.1	23.9	153.0	17.6
140.0 150.0	80.9 80.6	30.1	173.0 173.0	20.2
160.0	81.4	36.8	174.0	21.2
170.0	80.3	49.5	212.0	24.5
190.0	81.2 81.8	52.2 56.9	207.0	∠5.4 25.6
200.0	81.5	62.1	219.0	25.0

(8) first by area, fill by coefficient, with backup

num. panes	effic.	CPU time	back ups	off end
50.0	77.0	4.43	83.4	9.40
60.0	78.5	5.46	89.6	10.4
70.0	79.1	8.41	114.0	12.8
80.0	79.5	9.40	111.0	12.5
90.0	81.2	11.3	118.0	11.9
100.0	79.9	14.5	132.0	15.2
110.0	81.3	19.7	161.0	14.8
120.0	82.6	18.4	143.0	12.6
130.0	82.3	20.5	154.0	16.9
140.0	82.2	24.7	177.0	18.0
150.0	80.9	30.2	186.0	21.5
160.0	82.3	35.1	192.0	19.1
170.0	81.8	33.5	186.0	21.5
180.0	82.1	39.3	206.0	21.5
190.0	82.5	38.8	220.0	24.5
200.0	81.8	43.5	213.0	23.7

(23)	first by four-sort,		
	fill by coefficient,		
	NO backup		

num.		CPU
panes	effic.	time
50.0	71.5	1.16
60.0	71.7	1.81
70.0	72.5	2.67
80.0	73.1	3.49
90.0	72.1	4.91
100.0	74.0	6.07
110.0	73.8	7.63
120.0	74.5	9.34
130.0	75.6	11.1
140.0	75.0	14.0
150.0	74.8	16.5
160.0	75.1	19.3
170.0	74.7	23.1
180.0	75.4	25.2
190.0	75.9	29.3
200.0	75.1	33.4

(24) first by area, fill by coefficient, NO backup

num.		CPU
panes	effic.	time
50.0	69.4	1.13
60.0	70.9	1.53
70.0	71.8	2.18
80.0	71.1	2.77
90.0	71.7	3.76
100.0	72.0	4.56
110.0	72.9	5.70
120.0	72.8	6.92
130.0	73.7	8.14
140.0	73.7	9.78
150.0	73.2	11.2
160.0	74.0	12.8
170.0	73.0	14.7
180.0	73.5	16.3
190.0	74.1	18.7
200.0	73.8	20.3

(9) first by longest side, fill by four-sort, with backup

num.	effic.	CPU	back	off
panes		time	ups	end
50.0	79.5	2.83	39.0	5.70
60.0	79.2	4.75	55.8	9.20
70.0	79.8	6.08	55.8	9.20
80.0	80.9	7.64	58.2	7.90
90.0	80.4	11.1	70.3	11.7
100.0	81.4	14.1	81.5	11.9
110.0	82.5	18.1	87.8	9.80
120.0	83.0	21.7	91.0	10.0
130.0	83.0	25.6	99.0	12.2
140.0	83.9	27.5	89.4	11.3
150.0	82.8	36.3	116.0	12.5
160.0	83.0	41.7	119.0	14.1
170.0	82.7	49.3	126.0	15.8
180.0	83.8	52.4	114.0	12.4
200.0	82.5	04.2	143.0	19.6
	83.4	74.6	146.0	16.7

(10) first by coefficient, fill by four-sort, with backup

num.	effic.	CPU	back	off
panes		time	ups	end
50.0	67.9	4.86	61.1	14.9
60.0	66.3	7.52	82.9	19.7
70.0	67.7	8.40	64.4	21.1
80.0	67.5	10.8	71.7	23.7
90.0	68.1	16.3	102.0	26.1
100.0	69.0	19.4	101.0	28.6
110.0	69.8	22.5	90.3	31.5
120.0	69.8	27.7	114.0	34.4
130.0	72.0	32.6	123.0	32.7
140.0	73.3	43.7	145.0	33.4
150.0	71.0	46.2	133.0	41.2
160.0	70.4	50.2	135.0	43.1
170.0	73.3	61.1	150.0	40.6
190.0	72.1	79.5	178.0	48.6
200.0	13.9	91.0	196.0	4/.⊥

(25) first by longest side fill by four-sort, NO backup

num.		CPU
panes	effic.	time
50.0	74.5	1.14
60.0	73.2	1.68
70.0	73.6	2.47
80.0	74.4	3.30
90.0	73.9	4.52
100.0	74.3	5.93
110.0	75.0	7.70
120.0	76.4	9.81
130.0	76.1	11.8
140.0	76.6	14.1
150.0	76.2	17.1
160.0	76.4	20.4
170.0	76.0	23.4
180.0	77.0	27.0
190.0	77.7	32.1
200.0	76.4	36.4

(26) first by coefficient, fill by four-sort, NO backup

num. panes	effic.	CPU time
50.0 60.0 70.0 80.0 90.0 110.0 120.0 130.0 140.0 150.0 160.0 170.0 180.0	63.3 61.5 65.3 64.3 63.7 64.4 65.5 66.4 66.9 66.6 66.8 67.8 67.6 67.7	1.09 1.75 2.38 3.03 4.36 5.52 6.68 8.31 9.97 12.4 14.1 16.3 19.6 21.8
190.0 200 0	68.5 68 0	24.8 29 0
200.0	00.0	

(11) first by four-sort, fill by four-sort, with backup

num.	effic.	CPU	back	off
panes		time	ups	end
50.0	77.5	2.44	47.6	7.80
60.0	76.2	4.09	65.6	11.8
70.0	77.6	4.98	68.1	12.0
80.0	78.5	6.26	76.5	11.2
90.0	77.8	9.10	94.6	14.3
100.0	79.4	10.8	93.1	14.9
110.0	79.8	13.5	99.5	14.4
120.0	80.3	16.1	110.0	14.8
130.0	79.9	20.4	126.0	20.2
140.0	80.9	23.5	122.0	18.3
150.0	81.5	27.6	122.0	16.7
160.0	80.9	33.3	143.0	20.2
170.0	81.8	36.0	139.0	19.3
180.0	81.0 82.2	39.8 50.6	147.0 155.0	20.4
200.0	01.0	0.00	T00.0	ZI./

(12) first by area, fill by four-sort, with backup

num.	effic.	CPU	back	off
panes		time	ups	end
50.0	82.1	2.61	39.5	4.40
60.0	81.1	4.18	58.3	7.20
70.0	81.8	5.39	63.3	7.10
80.0	81.9	6.31	67.4	7.30
90.0	82.4	9.63	81.9	8.60
100.0	82.6	10.9	86.7	9.80
110.0	82.2	14.7	105.0	12.4
120.0	83.1	17.8	106.0	9.90
130.0	82.9	21.0	120.0	13.9
140.0	83.9	24.5	120.0	11.1
150.0	83.7	27.8	130.0	$11.9 \\ 11.7$
160.0	83.7	32.3	133.0	
170.0	83.4	34.0	137.0	14.6
180.0	84.2	40.9	149.0	14.0
190.0	84.2	50.1	$160.0 \\ 171.0$	11.8
200.0	84.2	52.4		15.2

(27) first by four-sort, fill by four-sort, NO backup

num. panes	effic.	CPU time
50.0 60.0 70.0 80.0 90.0 100.0 110.0 120.0 130.0 140.0 150.0 160.0 170.0 180.0 190.0	70.1 71.2 71.5 72.3 71.2 73.8 73.6 73.7 75.0 75.7 74.8 74.8 74.8 75.5 74.7 76.1 76.1	0.942 1.45 2.16 2.87 4.15 5.32 7.25 8.88 10.8 13.0 16.1 19.4 22.7 25.9 31.3 25

(28) first by area, fill by four-sort, NO backup

num.		CPU
panes	effic.	time
50.0	75.7	1.12
60.0	76.8	1.63
70.0	77.0	2.46
80.0	75.9	3.13
90.0	75.6	4.35
100.0	77.7	5.61
110.0	76.7	7.20
120.0	77.8	8.56
130.0	78.7	10.3
140.0	79.4	12.9
150.0	77.9	15.6
160.0	78.8	17.5
170.0	78.3	21.4
180.0	78.1	24.0
190.0	79.1	26.9
200.0	78.	30.8

(13) first by longest side, fill by area, with backup

num. panes	effic.	CPU time	back ups	off end
50.0 60.0 70.0 80.0 90.0 100.0 110.0 120.0 130.0 140.0 150.0 160.0 170.0 180.0 190.0	83.3 83.4 86.8 86.2 87.3 85.8 87.8 86.9 86.9 87.4 87.9 87.4 87.9 89.1 88.3 88.6 88.0	1.38 1.89 2.20 2.83 3.75 4.72 5.77 6.86 8.25 9.33 10.6 11.7 13.6 15.0 17.7 10.0	13.5 17.5 11.1 15.0 12.9 22.6 14.9 21.3 21.2 28.1 20.6 18.6 21.2 24.0 29.2	3.40 4.90 2.40 3.40 2.80 5.40 3.30 4.90 6.00 5.30 3.50 2.90 4.80 3.90 6.00
200.0	88.9	10.9	20.0	4.30

(14) first by coefficient, fill by area, with backup

num.	effic.	CPU	back	off
panes		time	ups	end
50.0	70.1	1.12	3.70	10.2
60.0	70.1	2.00	13.5	13.3
70.0	71.1	2.25	5.20	15.1
80.0	70.7	2.91	7.90	16.8
90.0	70.3	3.49	0.20	20.2
100.0	73.3	4.35	5.60	18.3
110.0	72.1	5.55	6.80	21.7
120.0	72.9	6.65	8.10	25.1
130.0	75.6	7.28	5.60	23.1
140.0	75.2	8.04	2.10	24.6
150.0	74.5	9.52	1.70	28.0
160.0	75.7	10.4	2.40	24.9
170.0	75.5	12.1	1.30	29.9
180.0	76.9	13.4	0.50	28.4
190.0	77.8	15.1	8.20	28.4
200.0	76.0	17.4	7.40	34.5

(29) first by longest side fill by area, NO backup

num.		CPU
panes	effic.	time
50.0	80.1	0.962
60.0	81.5	1.32
70.0	83.5	1.86
80.0	84.5	2.38
90.0	85.0	3.18
100.0	84.5	3.89
110.0	85.6	5.04
120.0	85.9	5.94
130.0	85.1	7.08
140.0	86.4	8.29
150.0	87.1	9.70
160.0	87.6	10.9
170.0	87.3	12.5
180.0	87.6	13.9
190.0	87.2	16.0
200.0	87.8	17.5

(30) first by coefficient, fill by area, NO backup

num. panes	effic.	CPU time
50.0 60.0 70.0 80.0 90.0 100.0 110.0 120.0 130.0 140.0 150.0 160.0 170.0	69.5 68.9 70.2 69.3 70.3 73.0 71.0 72.1 75.4 74.8 74.2 75.3 75.4	0.799 1.17 1.61 2.08 2.69 3.31 4.19 5.01 5.78 6.71 7.80 8.89 10.2
100.0	77.0	14.2
190.0	//.1	14.0
200.0	75.8	15.5

(15) first by four-sort, fill by area, with backup

num.	effic.	CPU	back	off
panes		time	ups	end
50.0	80.4	1.61	16.9	5.20
60.0	78.1	2.22	21.0	7.80
70.0	82.0	3.16	27.0	6.20
80.0	83.4	3.89	27.6	6.10
90.0	83.6	5.08	27.4	6.80
100.0	83.2	6.01	22.0	7.70
110.0	85.0	8.15	33.0	6.40
120.0	83.5	9.88	39.5	9.40
130.0	85.4	11.5	37.1	8.60
140.0	85.6	12.9	31.2	8.10
150.0	85.5	15.8	38.7	8.20
160.0	86.6	17.9	35.9	8.10
170.0	86.1	19.8	32.3	9.60
180.0	85.8	23.2	46.0	10.1
190.0	85.5	30.5	52.1	11.8
200.0	87.0		49.6	10.3

(16) first by area, fill by area, with backup

num.	effic.	CPU	back	off
panes		time	ups	end
$\begin{array}{c}\\ 50.0\\ 60.0\\ 70.0\\ 80.0\\ 90.0\\ 100.0\\ 110.0\\ 120.0\\ 130.0\\ 140.0\\ 150.0\\ 160.0\\ 170.0\\ 180.0 \end{array}$	85.4 85.2 87.6 86.9 87.6 86.7 88.7 88.2 88.5 88.8 88.8 89.7 89.0 88.8	1.85 2.72 3.80 4.95 6.96 9.37 11.9 14.8 18.7 23.3 28.2 33.5 39.6 46.3	22.6 31.6 26.0 29.2 35.6 45.1 31.6 37.8 40.0 37.2 37.7 34.2 40.9 46.4	$\begin{array}{c} 2.40 \\ 4.10 \\ 2.20 \\ 3.20 \\ 3.10 \\ 5.40 \\ 2.70 \\ 4.00 \\ 4.10 \\ 3.30 \\ 3.50 \\ 2.30 \\ 3.70 \\ 5.10 \end{array}$
190.0	89.6	55.0	34.4	3.10
200 0	89.9	62 4	36.8	2.90
200.0	02.2	02.1	50.0	2.70

(31) first by four-sort, fill by area, NO backup

50.0 77.4 1.1 60.0 75.7 1.6 70.0 80.2 2.3 80.0 80.9 3.1 90.0 81.3 4.2 100.0 80.4 5.1 110.0 82.0 6.5 120.0 81.8 7.6 130.0 82.7 9.3 140.0 83.5 10. 150.0 82.6 13. 160.0 84.3 15. 170.0 84.9 17. 180.0 82.7 20. 190.0 83.0 23.	10 64 312 23 15 59 9 7 6 4 7

(32) first by area, fill by area, NO backup

num.		CPU
panes	effic.	time
50.0	80.8	1.33
60.0	81.5	2.02
70.0	82.4	3.17
80.0	83.6	4.28
90.0	83.4	6.09
100.0	83.4	8.08
110.0	84.1	10.9
120.0	85.1	13.8
130.0	86.1	17.3
140.0	85.5	21.9
150.0	85.1	26.7
160.0	85.8	32.0
170.0	85.7	37.8
180.0	86.3	44.3
190.0	86.6	52.8
200.0	86.7	60.1

DATA SET TWO

(1) first by longest side, fill by longest side, with backup

num. panes	effic.	CPU time	back ups	off end
50.0 60.0 70.0 80.0 90.0 100.0 110.0 120.0 130.0 140.0 150.0 160.0 170.0 180.0 190.0	80.7 79.4 82.8 83.1 83.8 83.4 84.4 84.1 84.8 84.8 85.2 86.0 86.6 85.2 85.3	$\begin{array}{c} 1.33\\ 1.77\\ 2.65\\ 3.36\\ 4.98\\ 6.09\\ 8.46\\ 10.0\\ 12.6\\ 15.5\\ 18.7\\ 21.8\\ 25.3\\ 28.7\\ 34.1 \end{array}$	$\begin{array}{c} 5.90\\ 7.00\\ 7.70\\ 8.40\\ 9.90\\ 9.10\\ 12.3\\ 10.9\\ 11.9\\ 12.9\\ 13.1\\ 15.4\\ 14.2\\ 17.8\\ 15.5 \end{array}$	7.60 10.7 9.20 9.10 8.90 11.8 11.9 13.1 13.5 13.3 12.9 11.3 11.2 15.8 17.3
200.0	85.7	38.6	18.2	18.8

(4) first by area, fill by longest side, with backup

num.	effic.	CPU	back	off
panes		time	ups	end
50.0	76.5	1.22	2.70	8.70
60.0	78.0	1.57	2.90	10.7
70.0	80.0	2.06	0.800	8.50
80.0	81.6	2.73	6.40	10.7
90.0	81.5	3.53	4.20	10.9
100.0	80.5	4.34	3.40	14.1
110.0	83.8	5.30	4.80	11.0
120.0	82.0	6.34	4.60	16.0
130.0	82.5	7.33	3.00	15.3
140.0	82.4	8.88	7.10	19.0
150.0	84.1	9.88	5.50	14.7
160.0	85.0	10.9	6.40	11.2
170.0	85.2	12.6	6.60	14.2
190.0	84.3 84 3	13.9	6.00 5.70	1/.1
200.0	84.6	17.9	10.0	20.1

(17) first by longest side, fill by longest side, NO backup

num. panes	effic.	CPU time
50.0 60.0 70.0 80.0 90.0 100.0 110.0 120.0 130.0 140.0 150.0 160.0 170.0 180.0 190.0	78.1 78.7 81.1 81.1 81.3 81.7 81.6 82.5 82.6 83.3 83.3 83.3 84.2 84.5 83.9 83.7	$\begin{array}{c} 1.00\\ 1.44\\ 2.29\\ 3.01\\ 4.44\\ 5.57\\ 7.70\\ 9.34\\ 11.9\\ 14.4\\ 17.4\\ 20.6\\ 24.7\\ 28.0\\ 33.0\\ \end{array}$
200.0	03.0	20.0

(20) first by area, fill by longest side, NO backup

num.		CPU
panes	effic.	time
50.0	75.9	0.906
60.0	77.7	1.21
70.0	80.1	1.75
80.0	80.8	2.25
90.0	80.9	3.00
100.0	79.5	3.66
110.0	83.5	4.65
120.0	81.9	5.43
130.0	82.5	6.49
140.0	82.4	7.71
150.0	83.5	8.97
160.0	84.7	10.1
170.0	84.9	11.6
180.0	84.2	12.8
190.0	84.3	14.8
200.0	84.3	16.2
(13) first by longest side, fill by area, with backup

num. panes	effic.	CPU time	back ups	off end
50.0 60.0 70.0 80.0 90.0 100.0 110.0 120.0 130.0 140.0 150.0 160.0 170.0 180.0 190.0 200.0	81.3 81.2 83.6 84.7 84.8 83.2 85.6 84.8 84.4 85.1 85.9 87.4 85.9 85.4 85.0 86.6	$\begin{array}{c} 1.17\\ 1.49\\ 2.06\\ 2.46\\ 3.47\\ 4.09\\ 5.34\\ 6.29\\ 7.42\\ 8.68\\ 9.94\\ 11.0\\ 12.8\\ 14.2\\ 16.4\\ 17.8 \end{array}$	$\begin{array}{c} 2.80\\ 2.20\\ 3.00\\ 2.40\\ 4.20\\ 2.70\\ 3.90\\ 5.70\\ 3.30\\ 5.90\\ 2.80\\ 4.60\\ 3.30\\ 4.60\\ 3.30\\ 4.50\\ 5.70\end{array}$	$\begin{array}{c} 6.10\\ 7.70\\ 5.90\\ 5.50\\ 7.00\\ 9.20\\ 8.00\\ 10.4\\ 12.1\\ 10.5\\ 10.8\\ 8.90\\ 11.5\\ 13.6\\ 16.7\\ 13.7 \end{array}$

(16) first by area, fill by area, with backup

num. panes	effic.	CPU time	back ups	off end
50.0 60.0 70.0 80.0 90.0 100.0 110.0 120.0 130.0 140.0 150.0 160.0 170.0 180.0	80.3 78.9 82.2 82.3 82.9 83.9 84.4 83.6 85.2 84.7 86.0 86.4 86.2 86.4 86.0	1.42 1.97 2.92 3.82 5.54 6.83 9.51 11.8 14.7 18.0 21.4 25.5 30.0 34.4 41 1	4.90 5.40 5.30 5.80 7.70 5.70 10.8 8.70 11.4 9.80 10.1 12.1 11.8 12.3 14.3	6.80 10.5 7.40 7.00 8.80 9.50 10.1 12.8 12.1 12.5 11.3 9.40 10.3 10.8 14.6
200.0	86.4	46.6	14.4	16.4

(29) first by longest side, fill by area, NO backup

num. panes	effic.	CPU time
50.0 60.0 70.0 80.0 90.0 100.0 110.0 120.0 130.0 140.0 150.0 160.0 170.0 180.0 190.0	80.2 80.6 82.5 84.0 83.9 82.3 85.2 84.2 84.2 84.2 84.2 84.7 85.8 87.2 85.8 85.3 84.8 85.3 84.8	$\begin{array}{c} 0.945\\ 1.26\\ 1.82\\ 2.27\\ 3.08\\ 3.76\\ 4.88\\ 5.80\\ 6.87\\ 8.18\\ 9.39\\ 10.6\\ 12.2\\ 13.6\\ 15.5\\ 17.0\end{array}$

(32) first by area, fill by area, NO backup

num.		CPU
panes	effic.	time
50.0	78.6	1.14
60.0	78.2	1.64
70.0	81.5	2.61
80.0	81.3	3.49
90.0	80.7	5.07
100.0	82.4	6.34
110.0	82.3	8.84
120.0	82.7	11.0
130.0	84.0	13.8
140.0	83.8	17.1
150.0	84.5	20.6
160.0	84.8	24.7
170.0	84.8	29.3
180.0	85.1	33.6
190.0	84.3	39.5
200.0	85.3	44.5

DATA SET THREE

(1) first by longest side, fill by longest side, with backup

num. panes	effic.	CPU time	back ups	off end
50.0 60.0 70.0 80.0 90.0 100.0 110.0 120.0 130.0 140.0 150.0 160.0 170.0 180.0 190.0 200.0	80.0 80.4 82.5 82.5 84.6 83.4 84.7 83.3 84.2 84.8 85.7 86.2 86.2 85.7 85.5 86.0	1.47 1.95 2.74 3.62 5.27 6.54 8.78 10.8 13.4 16.1 19.6 22.6 26.9 30.5 36.9 40.1	$ \begin{array}{c} 16.1\\ 19.3\\ 18.2\\ 23.6\\ 25.3\\ 28.3\\ 30.7\\ 34.7\\ 39.9\\ 36.0\\ 41.3\\ 42.2\\ 39.7\\ 49.3\\ 53.5\\ 58.2\\ \end{array} $	9.10 10.1 8.20 10.4 7.90 13.3 10.5 16.2 15.7 13.3 12.3 11.2 12.0 15.4 18.0
			• -	

(4) first by area, fill by longest side, with backup

num. panes	effic.	CPU time	back ups	off end
50.0 60.0 70.0 80.0 90.0 100.0 110.0 120.0 130.0 140.0 150.0 160.0 170.0	76.8 78.7 80.1 81.2 81.4 80.1 83.6 82.0 82.5 82.6 83.9 85.0 85.2	1.22 1.61 2.07 2.82 3.70 4.55 5.48 6.43 7.43 9.01 10.2 11.1 12.9	4.50 4.80 2.30 14.2 10.9 9.90 11.9 9.80 8.40 13.7 15.3 15.5 19.1	8.20 10.1 9.00 10.5 11.5 15.3 12.0 16.1 16.0 17.9 14.6 10.7 13.7
180.0	84.4	14.1	15.3	17.5
190.0	84.4	16.5	15.6	19.4
200.0	84.8	18.6	27.7	20.8

(17) first by longest side, fill by longest side, NO backup

num. panes	effic.	CPU time
50.0 60.0 70.0 80.0 90.0 100.0 110.0 120.0 130.0 140.0 150.0 160.0 170.0 180.0 190.0 200.0	76.9 78.0 80.3 80.5 80.4 80.8 81.3 81.3 81.6 82.4 82.4 82.4 82.4 82.8 83.3 83.2 83.1	0.960 1.38 2.20 2.85 4.24 5.37 7.43 9.13 11.5 14.6 18.2 21.1 25.0 28.2 33.3 37.2
20010	00.1	J E

(20) first by area, fill by longest side, NO backup

num.		CPU
panes	effic.	time
50.0	76.1	0.901
60.0	77.8	1.22
70.0	80.0	1.75
80.0	79.6	2.26
90.0	80.3	3.00
100.0	79.6	3.68
110.0	82.2	4.68
120.0	81.4	5.45
130.0	82.2	6.49
140.0	81.9	7.78
150.0	82.7	9.04
160.0	83.8	10.2
170.0	83.7	11.8
180.0	83.9	12.9
190.0	83.5	14.9
200.0	83.6	16.4

(13) first by longest side, fill by area, with backup

num.	effic.	CPU	back	off
panes		time	ups	end
50.0	82.1	1.32	11.9	5.70
60.0	83.1	1.67	12.4	6.70
70.0	84.9	2.26	15.4	5.40
80.0	85.9	2.73	15.3	4.60
90.0	85.5	3.93	21.3	6.40
100.0	85.6	4.60	22.2	8.30
110.0	86.6	6.05	23.8	6.20
120.0	86.1	7.00	27.6	9.70
130.0	86.0	8.42	34.7	11.9
140.0	87.1	9.47	28.1	7.30
150.0	87.2	11.1	32.5	8.90
160.0	88.4	12.3	31.5	6.50
170.0	88.1	14.0	30.6	7.30
180.0	87.7	15.6	38.1	10.5
190.0	87.7	17.9	39.1	11.1
200.0	88.0	20.0	49.0	12.1

(16) first by area, fill by area, with backup

num.	effic.	CPU	back	off
panes		time	ups	end
$\begin{array}{c}\\ 50.0\\ 60.0\\ 70.0\\ 80.0\\ 90.0\\ 100.0\\ 110.0\\ 120.0\\ 130.0\\ 140.0\\ 150.0\\ 160.0\\ 170.0\\ 180.0 \end{array}$	80.3 79.6 81.6 82.5 82.7 82.9 84.1 83.2 84.6 84.1 85.9 86.2 85.6 85.8	1.60 2.19 3.16 4.17 6.15 7.74 10.6 13.0 16.1 19.4 24.1 27.9 33.0 38.0	13.8 14.2 15.0 17.1 19.7 20.7 29.0 27.6 33.8 28.0 35.4 35.0 35.5 38.4	$\begin{array}{c} 6.70\\ 9.20\\ 7.40\\ 7.80\\ 8.10\\ 10.6\\ 9.70\\ 13.6\\ 12.5\\ 12.9\\ 10.4\\ 10.2\\ 10.9\\ 12.0 \end{array}$
190.0	86.0	44.8	45.5	14.7
	86.2	52.5	46.3	14.9

(29) first by longest side, fill by area, NO backup

num. panes	effic.	CPU time
50.0 60.0 70.0 80.0 90.0 100.0 110.0 120.0 130.0 140.0 150.0 160.0 170.0 180.0 190.0 200.0	79.5 81.2 82.5 83.7 82.2 83.5 83.0 84.2 84.2 84.5 84.5 85.6 85.6 85.6 85.6 85.6 85.7	0.956 1.28 1.85 2.35 3.20 3.89 5.19 6.03 7.17 8.54 9.87 11.2 12.8 14.2 16.1 17.9

(32) first by area, fill by area, NO backup

num.		CPU
panes	effic.	time
50.0	77.6	1.17
60.0	78.1	1.65
70.0	80.3	2.68
80.0	80.5	3.58
90.0	80.4	5.19
100.0	81.1	6.65
110.0	81.7	9.17
120.0	81.6	11.3
130.0	82.4	14.2
140.0	82.3	17.5
150.0	83.5	21.6
160.0	83.7	25.3
170.0	83.2	29.8
180.0	83.8	34.8
190.0	83.1	40.6
200.0	84.3	46.2

APPENDIX E: data correlations

DATA SET ONE

```
( 1) first by longest side,
          fill by longest side,
          with backup
_____
Correlation of panes vs. CPU time (in seconds)
Equation:Slope (M):Intercept (B):Correlation Coeff.:Y = M * X + B0.3586942-24.164270.9578905log(Y) = M * X + B0.2276725E-01-0.27635360.9906324Y = M * log(X) + B37.33375-156.66030.8922952
                                             37.33375
2.561979
I = M \times \log(X) + B2.561979-9.599675log(Y) = M \times \log(X) + B2.561979-9.599675l/Y = M \times \log(X) + B-0.34008681.750255log(Y) = M \times 1/X + B-239.91854.840390Y = M \times Xlog(X) + B0.6250626E-01-17.60836Ylog(Y) = M \times X + B1.446721-111.0224
                                                                                                                          0.9976836
                                                                                                                       -0.9306338
                                                                                                                       -0.9676403
                                                                                                                          0.9658548
                                                                                   -111.0224
                                                                                                                          0.9304749
 _____
Correlation of panes vs. efficiency
Equation : Slope (M) : Intercept (B) : Correlation Coeff. :
EquationStope (M) :Intercept (B) :Correlation CoeffY = M * X + B0.2442647E-0184.840450.8754544\log(Y) = M * X + B0.2792969E-034.4411090.8731284Y = M * \log(X) + B2.86311174.294170.9183881\log(Y) = M*\log(X) + B0.3277678E-014.3203330.91705141/Y = M * \log(X) + B-0.3751733E-030.1316189E-01-0.9174498\log(Y) = M * 1/X + B-3.2072254.506377-0.9293724Y = M * Xlog(X) + B0.4174494E-0285.337160.8657119Ylog(Y) = M * X + B0.1336397376.72830.8758900
 _____
 (2) first by coefficient,
          fill by longest side,
          with backup
Correlation of panes vs. CPU time (in seconds)
Equation : Slope (M) : Intercept (B) : Correlation Coeff. :
Y = M * X + B 0.1230118 -6.002722 0.9725151
LequationSlope (M)Intercept (B)Correlation coeffY = M * X + B0.1230118-6.0027220.9725151log(Y) = M * X + B0.1569289E-010.3974426E-010.9744347Y = M * log(X) + B13.08613-52.784480.9259257log(Y) = M*log(X) + B1.780104-6.4540190.98926001/Y = M * log(X) + B-0.35473091.867270-0.9041581log(Y) = M * 1/X + B-168.66033.597732-0.9707577Y = M * Xlog(X) + B0.2136389E-01-3.7101870.9772973Ylog(Y) = M * X + B0.3958172-26.653560.9495096
 _____
Correlation of panes vs. efficiency

Equation : Slope (M) : Intercept (B) : Correlation Coeff. :

Y = M * X + B 0.4448529E-01 67.49559 0.9513355

log(Y) = M * X + B 0.6112420E-03 4.214386 0.9502265

Y = M * log(X) + B 4.979292 49.40495 0.9530159

log(Y) = M*log(X) + B 0.6853429E-01 3.965258 0.9535364

1/Y = M * log(X) + B -0.9442147E-03 0.1818509E-01 -0.9535899

log(Y) = M * 1/X + B -6.373644 4.351118 -0.9184384

Y = M * Xlog(X) + B 0.7658675E-02 68.36583 0.9476929

Ylog(Y) = M * X + B 0.2352243 284.1300 0.9516212
                                                        ------
```

(3) first by four-sort, fill by longest side, with backup _____ Correlation of panes vs. CPU time (in seconds) Slope (M) :Intercept (B) :Correlation Coeff. :0.2636029-16.389120.96787390.2032963E-01-0.10788080.9862767 Equation : Y = M * X + Blog(Y) = M * X + BY = M * log(X) + B10g(1) = MX + B0.2032903E-01-0.107808Y = M * log(X) + B27.68512-114.9412log(Y) = M*log(X) + B2.298005-8.4820481/Y = M * log(X) + B-0.33223961.718543log(Y) = M * 1/X + B-216.40454.481600Y = M * Xlog(X) + B0.4586972E-01-11.53085Ylog(Y) = M * X + B1.000122-73.698680.9097673 0.9977818 -0.9248751 -0.9731588 0.9745215 0.9432706 _____ Correlation of panes vs. efficiency

 Equation :
 Slope (M) :
 Intercept (B) :
 Correlation Coeff. :

 Y = M * X + B
 0.2917693E-01
 78.77788
 0.7734522

 log(Y) = M * X + B
 0.3564166E-03
 4.367112
 0.7711412

 Y = M * log(X) + B
 3.361363
 66.45875
 0.7974889

 log(Y) = M*log(X) + B
 0.4111385E-01
 4.216375
 0.7961217

 l/Y = M * log(X) + B
 -0.5031857E-03
 0.1452785E-01
 -0.7946514

 4.210375 0.1452785E-01 -0.7946514 4 449253 -0.7964544 log(Y) = M * 1/X + B -3.971336Y = M * Xlog(X) + B 0.5002453E-024.449253 Y = M * Xlog(X) + B 0.5002453E-02 79.36133 Ylog(Y) = M * X + B 0.1577004 343.9559 0.7673150 0.7740521 _____ (4) first by area, fill by longest side, with backup _____ Correlation of panes vs. CPU time (in seconds) Equation : Slope (M) : Intercept (B) : Correlation Coeff. : Y = M * X + B 0.1477559 -7.846981 0.9762319 M = MM = M< 0.9801884 Y = M * log(X) + B15.67284-63.82242log(Y) = M*log(X) + B1.928328-7.0662941/Y = M * log(X) + B-0.36309111.898599log(Y) = M * 1/X + B-182.86013.823914Y = M * Xlog(X) + B0.2567297E-01-5.100459Ylog(Y) = M * X + B0.4937997-34.286510.9267684 0.9964057 -0.9190056 -0.9786044 0.9814794 0.9532875 _____ Correlation of panes vs. efficiency Equation : Slope (M) : Y = M * X + B 0.3166131E-01 Intercept (B) : Correlation Coeff. : Y = M * X + B0.3166131E-0182.536090.7541319log(Y) = M * X + B0.3706916E-034.4134790.7516101Y = M * log(X) + B3.82286068.335430.8149335log(Y) = M*log(X) + B0.4487898E-014.2466440.81440061/Y = M * log(X) + B-0.5271512E-030.1407147E-01-0.8100947log(Y) = M * 1/X + B-4.6009884.503365-0.8647273Y = M * Xlog(X) + B0.5390573E-0283.192400.7429339Ylog(Y) = M * X + B0.1724835364.22990.7547908Y = M * X + B0.3166131E-01 82.53609 0.7541319

(5) first by longest side, fill by coefficient, with backup ______ Correlation of panes vs. CPU time (in seconds) 0.3144458-14.81885Correlation Coef:0.1525754E-011.0692970.0015533.53124121 Equation : Slope (M) : Intercept (B) : Correlation Coeff. : Y = M * X + Bloq(Y) = M * X + B33.53124 1.731880 Y = M * log(X) + Blog(Y) = M*log(X) + B-5.249825 0.9972946 log(Y) = M * log(X) + B1.731880l/Y = M * log(X) + B-0.1272667log(Y) = M * 1/X + B-164.2558Y = M * Xlog(X) + B0.5458658E-01Ylog(Y) = M * X + B1.3118470.6717943 -0.9246487 4.531178 -0.9796234 -8.943718 0.9885994 -81.12220 0.9705025 _____ Correlation of panes vs. efficiency Equation : Slope (M) : Y = M * X + B 0.1242601E-01 Intercept (B) : Correlation Coeff. :

 LOG(Y) = M * X + B
 0.1242601E-01

 Y = M * log(X) + B
 0.1614775E-03

 LOG(Y) - M+1
 1.48471^E

 76.30300 0.4972233 0.1242601E-01 4.334569 0.5004347 Y = M * log(X) + B 1.484/15 log(Y) = M*log(X) +B 0.1929628E-01 1/Y = M * log(X) + B -0.2507065E-03 log(Y) = M * 1/X + B -1.849781 Y = M * Xlog(X) + B 0.2109978E-02 70.80396 4.263097 0.5317142 0.5352102 0.1403790E-01 -0.5321670 4.372262 76.56403 330.7617 -0.5313785 Y = M * Xlog(X) + BYlog(Y) = M * X + B0.4885335 0.6640258E-01 0.4971235 _____ (6) first by coefficient, fill by coefficient, with backup Correlation of panes vs. CPU time (in seconds) Equation: Slope (M): Intercept (B): Correlation Coeff.: Y = M * X + B 0.5184605 -31.35818 0.9692701 -31.35818 0.9692701 log(Y) = M * X + B Y = M * log(X) + B0.1943483E-01 54.51412 0.7350092 0.9867715 54.51412 -225.4890 0.9121232 2.195915 loq(Y) = M*loq(X) + B-7.266088 0.9978536 $\log(Y) = M * \log(X) + B$ -0.14930840.7743184 $\log(Y) = M * 1/X + B$ -206.90685.122744 $Y = M * X \log(X) + B$ 0.9020033E-01-21.79224 $Y \log(Y) = M * X + B$ 2.332592-164.9704-0.9185656 -0.9737759 0.9757400 -164.9704 0.9498693 _____ Correlation of panes vs. efficiency Equation: Slope (M): Intercept (B): Correlation Coeff.: Y = M * X + B 0 5166222E 01 70 27247 Y = M * X + B0.5166223E-0172.373470.9558959 $\log(Y) = M * X + B$ 0.6607307E-034.2842110.9525911Y = M * $\log(X) + B$ 5.95499850.545420.9861307 $\log(Y) = M*\log(X) + B$ 0.7633349E-014.0042240.9849461 $1/Y = M * \log(X) + B$ -0.9789919E-030.1734843E-01-0.9845573 $\log(Y) = M * 1/X + B$ -7.3089404.435982-0.9767528Y = M * X log(X) + B0.8849387E-0273.411600.9474308Y = M * Xlog(X) + B-7.3089404.435982Ylog(Y) = M * X + B0.2768787309.7103 0.9474308 309.7103 0.9564112 _____

(7) first by four-sort, fill by coefficient, with backup _____ Correlation of panes vs. CPU time (in seconds) Slope (M) : Intercept (B) : Correlation Coeff. : 0 3915176 -21 06845 0 9809112 Equation : Slope (M) Y = M * X + B 0.3915176 0.3915176 -21.06845 0.1756035E-01 0.8478143 0.9809112 log(Y) = M * X + B0.9780392 Y = M * log(X) + B-170.1339 -6.459882 0.7815449 Y = M * log(X) + B41.68588-170.13390.9347227log(Y) = M*log(X) + B2.000605-6.4598820.99724051/Y = M * log(X) + B-0.14997580.7815449-0.9190278log(Y) = M * 1/X + B-190.12714.842418-0.9815569Y = M * Xlog(X) + B0.6797741E-01-13.760320.9854618Ylog(Y) = M * X + B1.678887-110.79790.9662753_____ Correlation of panes vs. efficiency Equation : Slope (M) : Intercept (B) : Correlation Coeff. : Y = M * X + B 0.3191176E-01 75.76103 0.9115254 log(Y) = M * X + B 0.4019703E-03 4.328444 0.9129177 Y = M * log(X) + B 3.643568 62.44330 0.9314513 log(Y) = M*log(X) + B 0.4594352E-01 4.160462 0.9338503 1/Y = M * log(X) + B -0.5797218E-03 0.1529803E-01 -0.9307266 log(Y) = M * 1/X + B -4.339416 4.419764 -0.9135180 Y = M * Xlog(X) + B 0.5474082E-02 76.39749 0.9047447 Ylog(Y) = M * X + B 0.1715074 327.7950 0.9118733 _____ (8) first by area, fill by coefficient, with backup Correlation of panes vs. CPU time (in seconds) Equation:Slope (M):Intercept (B):Correlation Coeff.:Y = M * X + B0.2703472-11.468400.9896076log(Y) = M * X + B0.1478590E-011.0489580.9727198 log(Y) = M * X + BY = M * log(X) + B1.048958 -116.7505 Y = M * log(X) + B29.27947-116.7505log(Y) = M*log(X) +B1.689383-5.1272581/Y = M * log(X) + B-0.13254060.7015382log(Y) = M * 1/X + B-160.70714.418295Y = M * Xlog(X) + B0.4679984E-01-6.336742Ylog(Y) = M * X + B1.091154-63.25500 0.9592232 0.9946797 -0.9426922 -0.9799937 0.9912479 0.9827647 Correlation of panes vs. efficiency Correlation of panes vs. efficiencyEquation :Slope (M) :Y = M * X + B0.2835294E-01log(Y) = M * X + B0.3536880E-034.3492660.8108057log(Y) = M * X + B 0.3536880E-03 Y = M * log(X) + B 3.455132 log(Y) = M*log(X) + B 0.4315329E-01 1/Y = M * log(X) + B -0.5390135E-03 64.52586 4.188501 0.8848565 0.8853720 log(Y) = M * 1/X + B -4.368388Y = M * Xlog(X) + B 0.4811276E-02 77.99092 Ylog(Y) = M * X + B 0.1526581 336.5471 0.7966175 0.8115287 _____

```
( 9) first by longest side,
        fill by four-sort,
        with backup
Correlation of panes vs. CPU time (in seconds)
                                      Equation: Slope (M): Intercept (B): Correlation Coeff.:
Y = M * X + B 0.4545824 -28.20404 0.9696175
Y = M * X + B0.4545824-28.20404log(Y) = M * X + B0.2041396E-010.4278324Y = M * log(X) + B47.78109-198.3382log(Y) = M*log(X) + B2.312887-8.0064791/Y = M * log(X) + B-0.19850601.025196log(Y) = M * 1/X + B-218.42785.047005Y = M * Xlog(X) + B0.7909237E-01-19.81998Ylog(Y) = M * X + B1.971924-142.1985
                                                                                                         -0.9132135
                                                                                                       -0.9767868
                                                                                                          0.9761561
                                                                                                          0.9485312
 _____
Correlation of panes vs. efficiency
Correlation of panes vs. efficiencyEquation :Slope (M) :Intercept (B) : Correlation Coeff. :Y = M * X + B0.2808823E-0178.476470.8573615log(Y) = M * X + B0.3446691E-034.3633120.8574929Y = M * log(X) + B3.31851266.224790.9065636log(Y) = M*log(X) + B0.4075527E-014.2128110.90745961/Y = M * log(X) + B-0.5004281E-030.1457817E-01-0.9084968log(Y) = M * 1/X + B-3.9643994.443919-0.9142265Y = M * Xlog(X) + B0.4788326E-0279.054980.8457071Ylog(Y) = M * X + B0.1516967342.33520.8570177
 _____
 (10) first by coefficient,
        fill by four-sort,
        with backup
Correlation of panes vs. CPU time (in seconds)
Equation: Slope (M): Intercept (B): Correlation Coeff.:
Y = M * X + B 0.5551236 -32.53546 0.9749827
I - M × A + B0.5551236-32.535460.9749827log(Y) = M * X + B0.1875813E-010.93808320.9873103Y = M * log(X) + B58.57302-241.36290.9207041log(Y) = M*log(X) + B2.116685-6.7712640.0070022l/Y = M * log(X) + B0.10101000.1010100
log(Y) = M * log(X) + B2.116685-6.771264l/Y = M * log(X) + B-0.12181990.6344870log(Y) = M * 1/X + B-198.85755.165043Y = M * Xlog(X) + B0.9652132E-01-22.25779Ylog(Y) = M * X + B2.546058-175.7648
                                                                                                      -0.9409048
                                                                                                        -0.9701846
                                                                                                         0.9809056
                                                                                                         0.9583171
_____
Correlation of panes vs. efficiency
Equation :Slope (M) :Intercept (B) :Correlation Coeff. :Y = M * X + B0.4547059E-0164.603680.9051374
Equation :Slope (M) :Intercept (B) :Correlation CoeffY = M * X + B0.4547059E-0164.603680.9051374log(Y) = M * X + B0.6483801E-034.1710030.9056827Y = M * log(X) + B5.03146646.388390.8963834log(Y) = M*log(X) + B0.7187360E-013.9106560.89852691/Y = M * log(X) + B-0.1026857E-020.1912027E-01-0.9010555log(Y) = M * 1/X + B-6.5570714.314113-0.8489939Y = M * Xlog(X) + B0.7839146E-0265.486560.9029205Ylog(Y) = M * X + B0.2387518269.09840.9048719
                                                      -----
```

(11) first by four-sort, fill by four-sort, with backup _____ Correlation of panes vs. CPU time (in seconds) Correlation of panes vs. CPU time (in seconds)Equation :Slope (M) :Intercept (B) :Correlation Coeff. :Y = M * X + B0.3441780-20.892870.9719909log(Y) = M * X + B0.1968704E-010.28380200.9866779Y = M * log(X) + B36.22990-149.96020.9157177log(Y) = M*log(X) + B2.225919-7.8282830.99843471/Y = M * log(X) + B-0.23158831.200156-0.9222842log(Y) = M * 1/X + B-209.72044.729694-0.9742802Y = M * Xlog(X) + B0.5986699E-01-14.535110.9782804Ylog(Y) = M * X + B1.403994-100.83140.9509985_____ Correlation of panes vs. efficiency Equation : Slope (M) : Intercept (B) : Correlation Coeff. : Y = M * X + B 0.3598483E-01 75.32066 0.9412419 log(Y) = M * X + B 0.4526367E-03 4.322933 0.9387811 Y = M * log(X) + B 4.061677 60.52607 0.9508290 log(Y) = M*log(X) + B 0.5117660E-01 4.136427 0.9499514 1/Y = M * log(X) + B -0.6446144E-03 0.1559644E-01 -0.9502165 log(Y) = M * 1/X + B -4.778090 4.424737 -0.9185805 Y = M * Xlog(X) + B 0.6184601E-02 76.03111 0.9360310 Ylog(Y) = M * X + B 0.1934577 325.4242 0.9414668 _____ (12) first by area, fill by four-sort, with backup Correlation of panes vs. CPU time (in seconds) Correlation of panes vs. CPU time (in seconds)Equation :Slope (M) :Intercept (B) :Correlation Coeff. :Y = M * X + B0.3322676-19.375950.9780152log(Y) = M * X + B0.1905622E-010.38614440.9830638Y = M * log(X) + B35.17728-144.93220.9266916log(Y) = M*log(X) + B2.162276-7.5024920.99832351/Y = M * log(X) + B-0.21821461.132408-0.9242651log(Y) = M * 1/X + B-204.38474.702682-0.9773305Y = M * Xlog(X) + B0.5774058E-01-13.204700.9834122Ylog(Y) = M * X + B1.350606-94.450750.9594955_____ Correlation of panes vs. efficiency Correlation of panes vs. efficiencyEquation :Slope (M) :Intercept (B) : Correlation Coeff. :Y = M * X + B0.1929458E-0180.550670.9338064log(Y) = M * X + B0.2327378E-034.3892310.9285011Y = M * log(X) + B2.11851272.899700.9176305log(Y) = M*log(X) + B0.2556029E-014.2969140.9126337l/Y = M * log(X) + B-0.3086831E-030.1352145E-01-0.9178413log(Y) = M * 1/X + B-2.3133484.440219-0.8554699Y = M * Xlog(X) + B0.1045313353.49960.9338580 _____

(13) first by longest side, fill by area, with backup _____ Correlation of panes vs. CPU time (in seconds)

 U.IIXII47
 -6.359331
 Correlation Coef:

 0.1723779E-01
 -0.3025934

 12 55607

 Equation: Slope (M): Intercept (B): Correlation Coeff.: Y = M * X + B 0 1181147 -6 359331 0 9843534 Y = M * X + B log(Y) = M * X + BY = M * log(X) + B12.55607 0.9365187 -51.23550 Y = M + log(X) + B12.5300751.20000log(Y) = M*log(X) + B1.947551-7.3986091/Y = M + log(X) + B-0.44439012.330640log(Y) = M + 1/X + B-183.03743.584586Y = M + Xlog(X) + B0.2051109E-01-4.156656Ylog(Y) = M + X + B0.3653664-25.88904 0.9981698 -0.9556296 -0.9716017 0.9890831 0.9642349 Correlation of panes vs. efficiency 83.2002 4.422657 70.22050 4.271081 log(Y) = M * X + B 0.3480009E-03 Y = M * log(X) + B 3.540475 0.8404815 0.8837335 0.8877231 -0.8808535 log(Y) = M * 1/X + B-4.0676414.504657Y = M * Xlog(X) + B0.5135047E-0283.89265Ylog(Y) = M * X + B0.1640827368.24454.504657 -0.9106230 0.8286777 0.8386489 _____ (14) first by coefficient, fill by area, with backup _____ Correlation of panes vs. CPU time (in seconds) Equation: Slope (M): Intercept (B): Correlation Coeff.: Y = M * X + B 0.1032617 -5.310218 0.9838642
 0.1656501E-01
 -5.310218

 0.1656501E-01
 -0.2994744

 10.99348
 -44.62078

 1.880687
 -7.161002
 0.9838642 log(Y) = M * X + B Y = M * log(X) + B 0.9818166 0.9374458 log(Y) = M * log(X) + Blog(9348)log(Y) = M * log(X) + Bl.880687l/Y = M * log(X) + B-0.4793233log(Y) = M * 1/X + B-178.1354Y = M * Xlog(X) + B0.1792952E-01Ylog(Y) = M * X + B0.30931470.9976321 2.512493 3.457211 -0.9205407 -0.9786716 -3.383122 0.9884649 -21.66616 0.9620998 _____ Correlation of panes vs. efficiency Equation :Slope (M) :Intercept (B) :Correlation Coeff. :Y = M * X + B0.5167647E-0167.152950.9409599 67.152950.94095994.2103030.940578146.539380.92884773.9290430.9302192 log(Y) = M * X + BY = M * log(X) + B0.7033548E-03 5.699686 0.1863693E-01 -0.9320202 4.365389 -0.8796356 68.15250 0.9393116 282.2725 0.9408991

(15) first by four-sort, fill by area, with backup Correlation of panes vs. CPU time (in seconds) Equation : Slope (M) : Intercept (B) : Correlation Coeff. : -11.12426 Y = M * X + B0.1886441 0.9750718 log(Y) = M * X + B0.1892458E-01 -0.1716115 0.9877909 Y = M * log(X) + B19.89695 -82.05293 0.9204394 log(Y) = M*log(X) +B2.138189 -7.962295 0.9988501 1/Y = M * log(X) + B -0.3709849log(Y) = M * 1/X + B -201.2264 1.929552 -0.9382440 4.098577 -0.9735811 Y = M * Xlog(X) + B 0.3280280E-01 -7.633250 Ylog(Y) = M * X + B 0.6607307 -47.91335 0.9810724 -47.91335 0.9518071 _____ Slope (M) :Intercept (B) :Y = M * X + B0.4374954E-0178.70007log(Y) = M * X + B0.5252757E-034.366776Y = M * log(X) + B5.12684959.81657log(Y) = M*log(X) + B0.6167638E-014.139477l/Y = M * log(X) + B-0.7426427E-030.1541774E-log(Y) = M * 1/X + B-6.0165824.489383Y = M * Xlog(X) + B0.7474964E-0279.59084Ylog(Y) = M * X + B0.2372390343 4007 Correlation of panes vs. efficiency Equation:Slope (M):Intercept (B):Correlation Coeff.:Y = M * X + B0.4374954E-0178.700070.8601300 0.8691390 4.366776 59.81657 4.139477 0.8658842 0.9115523 0.9099274 -0.9033660 0.1541774E-01 -6.0165824.4893830.7474964E-0279.590840.2372390343.4827 -0.9193284 0.8592542 0.8701215 _____ (16) first by area, fill by area, with backup _____ Correlation of panes vs. CPU time (in seconds) Equation : Slope (M) : Intercept (B) : Correlation Coeff. : Y = M * X + B 0.3948926 -26.65219 0.9614317 -26.65219 -0.2279348 -172.7634 -9.690323 1.665543 4.962461 -19.42419 -124.4336 log(Y) = M * X + BY = M * log(X) + B0.2306437E-01

 I = M * log(X) + B
 41.15276

 log(Y) = M*log(X) + B
 2.599076

 1/Y = M * log(X) + B
 -0.3240909

 log(Y) = M * 1/X + B
 -243.7760

 Y = M * Xlog(X) + B
 0 68707107 21

 0.9901251 0.8967135 0.9985797 -0.9239740 -0.9700356 Y = M * Xlog(X) + B0.6879710E-01Ylog(Y) = M * X + B1.6265610.9691842 0.9365447 _____ Correlation of panes vs. efficiency Equation : Slope (M) : Intercept (B) : Correlation Coeff. : Y = M * X + B -1.441382 316.2603 -0.3588463 316.2603 5.134521 Y = M * X + B-1.441302510.2000log(Y) = M * X + B-0.4098518E-025.134521-0.3439529Y = M * log(X) + B-181.7804999.5320-0.4050341log(Y) = M*log(X) + B-0.51927397.088722-0.39001741/Y = M * log(X) + B0.2139793E-020.5309833E-030.3583711log(Y) = M * 1/X + B53.393094.1168380.4153417Y = M * Xlog(X) + B-0.2428463284.8145-0.3498304Y = M * Xlog(X) + B -0.2428463 284.8145 -0.3498304 Ylog(Y) = M * X + B -10.13739 1997.890 -0.3599912 _____

(17) first by longest side, fill by longest side, NO backup _____ Correlation of panes vs. CPU time (in seconds) Slope (M) :Intercept (B) :Correlation Coeff. :0.3236191-22.281770.96087750.2445771E-01-0.66984580.9868807 Equation : Y = M * X + Blog(Y) = M * X + BY = M * log(X) + B33.70749 -141.9377 0.8957267 1 - M10g(X) + B33.70743-141.9377log(Y) = M*log(X) + B2.767683-10.758941/Y = M * log(X) + B-0.49523692.532305log(Y) = M * 1/X + B-260.79144.855768Y = M * Xlog(X) + B0.5638441E-01-16.36102Ylog(Y) = M * X + B1.258965-97.939160.9994949 -0.9016739 -0.9754172 0.9687008 0.9334045 Correlation of panes vs. efficiency Correlation of panes vs. efficiencyEquation :Slope (M) :Intercept (B) : Correlation Coeff. :Y = M * X + B0.3807353E-0180.134560.9207097log(Y) = M * X + B0.4512580E-034.3847380.9235247Y = M * log(X) + B4.41739163.911440.9560505log(Y) = M*log(X) + B0.5244285E-014.1920460.96056171/Y = M * log(X) + B-0.6232972E-030.1474609E-01-0.9554379log(Y) = M * 1/Y + B-5.0921004.489343-0.9659815

 4.192040
 -0.9554379

 4.489343
 -0.9659815

 0.120384

 log(Y) = M * 1/X + B-5.0921004.489343Y = M * Xlog(X) + B0.6518035E-0280.90189Ylog(Y) = M * X + B0.2069430351.20080.9120384 0.9210185 _____ (18) first by coefficient, fill by longest side, NO backup _____ Correlation of panes vs. CPU time (in seconds)
 U.1024051
 -5.812453
 Correlation Coef:

 0.1839551E-01
 -0.6648906
 0.9802996

 10 85220
 -0.6648906
 0.9802996
 Equation: Slope (M): Intercept (B): Correlation Coeff.: Y = M * X + B 0.1024051 -5.812453 0.9802996 log(Y) = M * X + BY = M * log(X) + B 10.85320 -44.56381 0.9298448 $I = M \times 100(X) + B$ 10.85320-44.56381 $log(Y) = M \times log(X) + B$ 2.088178-8.284157 $1/Y = M \times log(X) + B$ -0.64585043.358080 $log(Y) = M \times 1/X + B$ -197.63353.505157 $Y = M \times Xlog(X) + B$ 0.1779301E-01-3.908829 $Ylog(Y) = M \times X + B$ 0.2982060-21.973470.9988809 -0.9197196 -0.9791286 0.9855607 0.9548009 _____ Correlation of panes vs. efficiency Intercept (B) : Correlation Coeff. : Equation : Slope (M) : 0.5461719E-01 65.37286 0.9531755 Y = M * X + B0.5461719E-0165.37286log(Y) = M * X + B0.7611156E-034.183624Y = M * log(X) + B6.17140342.88626log(Y) = M*log(X) + B0.8621694E-013.8692391/Y = M * log(X) + B-0.1205799E-020.1959674E-log(Y) = M * 1/X + B-8.1114094.355538Y = M * Xlog(X) + B0.2880368273.0196Y = M * X + B 4.183624 42.88626 0.9520835 0.9639240 0.9652335 0.1959674E-01 -0.9646217 4.355538 -0.9405221 0.9481108 0.9535429 _____

(19) first by four-sort, fill by longest side, NO backup Correlation of panes vs. CPU time (in seconds) Equation: Slope (M): Intercept (B): Correlation Coeff.: Y = M * X + B 0.2183985 -14.27794 0.9653799 Y = M * X + B log(Y) = M * X + B Y = M * log(X) + B log(Y) = M*log(X) + B 2.516282 -0.6436822 -95.56753 -9.819161 2.748103 -14.27794 0.9857349 0.9044070 0.9993494 1/Y = M * log(X) + B -0.5347213log(Y) = M * 1/X + B -237.4273 2.748103 -0.9128655

 Y = M * Xlog(X) + B
 -23/.4273
 4.380267

 Y = M * Xlog(X) + B
 0.3802348E-01
 -10.26494

 Ylog(Y) = M * X + B
 0.7760652
 -59.34378

 -0.9766111 0.9725171 -59.34378 0.9363085 _____ Correlation of panes vs. efficiency Equation: Slope (M): Intercept (B): Correlation Coeff.: Y = M * X + B 0.3352041F-01 75.05000 Y = M * X + B0.3352941E-01 75.95882 0.8398774 Y = M * X + B0.3352941E-01log(Y) = M * X + B0.4217601E-03Y = M * log(X) + B3.944778log(Y) = M*log(X) + B0.4969744E-011/Y = M * log(X) + B-0.6266240E-03log(Y) = M * 1/X + B-4.897451Y = M * Xlog(X) + B0.5726807E-024.330913 0.8398017 61.41257 4.147573 0.8843575 4.14/5/5 0.1545975E-01 0.8856470 -0.9039192 Y = M * Xlog(X) + B-4.8974514.429987Ylog(Y) = M * X + B0.1802500328.8596 0.8300405 328.8596 0.8402857 _____ (20) first by area, fill by longest side, NO backup Correlation of panes vs. CPU time (in seconds) ImplementImplementCorrelation CoefficientI - M ^ X + B0.1284941-7.4080170.9799967log(Y) = M * X + B0.1887876E-01-0.52531770.9816315Y = M * log(X) + B13.61800-56.030810.9295437log(Y) = M*log(X) + B2.146650-8.3619150.99896811/Y = M * log(X) + B-0.55699032.889182-0.9196364log(Y) = M * 1/X + B-203.47573.760433-0.9806996Y = M * Xlog(X) + B0.223256550.1-Equation: Slope (M): Intercept (B): Correlation Coeff.: Y = M * X + B 0.1284941 -7.408017 0.9799967 log(Y) = M * 1/X + B-203.47573.760433Y = M * Xlog(X) + B0.2232595E-01-5.019385Ylog(Y) = M * X + B0.4012252-29.288380.9852536 0.9561886 _____ Correlation of panes vs. efficiency Equation: Slope (M): Intercept (B): Correlation Coeff.: Y = M * X + B 0.4001471E-01 79.19191 0.8744949 Equation :Slope (M) :Intercept (B) :Correlation CoeffY = M * X + B0.4001471E-0179.191910.8744949log(Y) = M * X + B0.4801528E-034.3727810.8678941Y = M * log(X) + B4.71017561.820740.9212766log(Y) = M*log(X) + B0.5667862E-014.1635800.91689971/Y = M * log(X) + B-0.6821930E-030.1512545E-01-0.9145025log(Y) = M * 1/X + B-5.6211634.486004-0.9418065Y = M * Xlog(X) + B0.2170074346.14190.8755817

```
(21) first by longest side,
          fill by coefficient,
         NO backup
_____
Correlation of panes vs. CPU time (in seconds)

      Equation:
      Slope (M):
      Intercept (B):
      Correlation Coeff.:

      Y = M * X + B
      0.1494736
      -8.771696
      0.9795758

      log(Y) = M * X + B
      0.1916814E-01
      -0.4351460
      0.9845610

      W = M + Ler(X) + B
      15.914327
      65.20467
      0.9845610

log(Y) = M * X + BY = M * log(X) + B
Y = M * log(X) + B15.81437-65.20467log(Y) = M*log(X) + B2.174406-8.3674111/Y = M * log(X) + B-0.49263612.555645log(Y) = M * 1/X + B-205.49133.905855Y = M * Xlog(X) + B0.2597775E-01-5.997118Ylog(Y) = M * X + B0.4874483-35.61034
                                                                                                                            0.9275572
                                                                                                                            0.9995825
                                                                                                                        -0.9275993
-0.9783728
                                                                                                                           0.9850810
                                                                                                                            0.9562577
_____
Correlation of panes vs. efficiency
Correlation of panes vs. efficiencyEquation :Slope (M) :Intercept (B) :Correlation Coeff. :Y = M * X + B0.2895542E-0169.199330.8590830log(Y) = M * X + B0.4004481E-034.2376870.8574474Y = M * log(X) + B3.38485056.740950.8987936log(Y) = M*log(X) + B0.4686239E-014.0651500.89805121/Y = M * log(X) + B-0.6486634E-030.1682019E-01-0.8984544log(Y) = M * 1/X + B-4.5193744.330519-0.8969906Y = M * Xlog(X) + B0.4945847E-0269.789760.8490683Ylog(Y) = M * X + B0.1529228293.14620.8589200
 _____
 (22) first by coefficient,
          fill by coefficient,
         NO backup
Correlation of panes vs. CPU time (in seconds)
Correlation of panes vs. CPO time (in seconds)Equation :Slope (M) :Intercept (B) :Correlation Coeff. :Y = M * X + B0.2854471-19.303380.9632114log(Y) = M * X + B0.2371193E-01-0.65242910.9853888Y = M * log(X) + B29.79363-125.14030.8997769log(Y) = M*log(X) + B2.688156-10.457000.99979391/Y = M * log(X) + B-0.51372672.628795-0.8971743log(Y) = M * 1/X + B-253.90014.714736-0.9780292Y = M * Xlog(X) + B0.4971807E-01-14.071460.9707492Ylog(Y) = M * X + B1.080005-83.413440.9354816
Correlation of panes vs. efficiency
Equation :Slope (M) :Intercept (B) :Correlation Coeff. :Y = M * X + B0.4835340E-0165.455830.95554450.4835340E-0165.455830.9552462
Y = M * X + B0.4835340E-0165.455830.9555445log(Y) = M * X + B0.6794864E-034.1842250.9533162Y = M * log(X) + B5.42250145.743480.9590448log(Y) = M*log(X) + B0.7634469E-013.9065290.95862871/Y = M * log(X) + B-0.1075094E-020.1910771E-01-0.9597599log(Y) = M * 1/X + B-7.0755244.336131-0.9201595Y = M * Xlog(X) + B0.8318516E-0266.405470.9511884Ylog(Y) = M * X + B0.2545919273.49760.9557635
                                 -----
                                                                                                        _____
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(23) first by four-sort, fill by coefficient, NO backup _____ Correlation of panes vs. CPU time (in seconds) Correlation of panes vs. CPU time (in seconds)Equation :Slope (M) :Intercept (B) :Correlation Coeff. :Y = M * X + B0.2116235-13.391690.9732741log(Y) = M * X + B0.2130509E-01-0.49457100.9842113Y = M * log(X) + B22.25601-92.653280.9160809log(Y) = M*log(X) + B2.418103-9.3172620.99975851/Y = M * log(X) + B-0.48850922.514332-0.9062778log(Y) = M * 1/X + B-228.73384.333540-0.9794530Y = M * Xlog(X) + B0.3681413E-01-9.4849160.9796762Ylog(Y) = M * X + B0.7498316-56.203430.9488114 _____ Correlation of panes vs. efficiency Equation : Slope (M) : Intercept (B) : Correlation Coeff. : 0.2714752E-01 70.65656 0.8968030 Y = M * X + Blog(Y) = M * X + BY = M * log(X) + B0.2714752E-010.3684513E-033.1483644.258505 0.8945685 59.09548 4.101396 0.9308231 Y = M * log(X) + B 3.148364 log(Y) = M*log(X) + B 0.4277230E-01 1/Y = M * log(X) + B -0.5810788E-03 log(Y) = M * 1/X + B -4.110488 Y = M * Xlog(X) + B 0.4644696E-02 Ylog(Y) = M * X + B 0.1438787 0.9294188 0.1626932E-01 -0.9310606 4.343468 -0.9250712 4.343468 71.20544 0.8878118 300.7943 0.8970031 _____ (24) first by area, fill by coefficient, NO backup _____ Correlation of panes vs. CPU time (in seconds) Equation: Slope (M): Intercept (B): Correlation Coeff.: Y = M * X + B 0.1299103 -7.459417 0.9838006

 Iog(Y) = M * X + B
 0.1299103
 -7.459417
 0.9838006

 Y = M * log(X) + B
 13.78864
 -56 71572

 13.78864 2.134121

 log(Y) = M*log(X) + B
 2.134121
 -8.285501
 0.9996667

 l/Y = M * log(X) + B
 -0.5304383
 2.756303
 -0.9342000

 log(Y) = M * 1/X + B
 -201.6520
 3.760077
 -0.9782990

 Y = M * Xlog(X) + B
 0.2256476E-01
 -5.040018
 0.9887602

 Ylog(Y) = M * X + B
 0.4066458
 -29.55182
 0.9625862

 _____ Correlation of panes vs. efficiency Equation : Slope (M) : Intercept (B) : Correlation Coeff. : Y = M * X + B 0.2482307E-01 69.49712 0.8856261 log(Y) = M * X + B 0.3440085E-03 4.241804 0.8858883 Y = M * log(X) + B 2.923532 58.71342 0.9335082 log(Y) = M*log(X) + B 0.4057597E-01 4.092072 0.9351776 1/Y = M * log(X) + B -0.5633509E-03 0.1645442E-01 -0.9321765 log(Y) = M * 1/X + B -3.987863 4.322550 -0.9519153 Y = M * Xlog(X) + B 0.4238019E-02 70.00451 0.8748922 Ylog(Y) = M * X + B 0.1310404 294.7200 0.8859951
 294.7200
 0.8850051

(25) first by longest side, fill by four-sort, NO backup _____ Correlation of panes vs. CPU time (in seconds) Correlation of panes vs. CPU time (in seconds)Equation :Slope (M) :Intercept (B) :Correlation Coeff. :Y = M * X + B0.2298956-15.058830.9681042log(Y) = M * X + B0.2236542E-01-0.61805200.9862462Y = M * log(X) + B24.08100-100.70500.9075722log(Y) = M*log(X) + B2.532855-9.8532670.99961781/Y = M * log(X) + B-0.51549052.648768-0.9143136log(Y) = M * 1/X + B-238.86474.438490-0.9763567Y = M * Xlog(X) + B0.4001908E-01-10.830850.9751137Ylog(Y) = M * X + B0.8270259-63.119500.9410953_____ Correlation of panes vs. efficiency Equation : Slope (M) : Intercept (B) : Correlation Coeff. : Y = M * X + B 0.2504366E-01 72.35080 0.8858852 log(Y) = M * X + B 0.3323472E-03 4.282192 0.8881660 Y = M * log(X) + B 2.767548 62.33559 0.8761737 log(Y) = M*log(X) + B 0.3674361E-01 4.149206 0.8788188 1/Y = M * log(X) + B -0.4881062E-03 0.1557076E-01 -0.8761848 log(Y) = M * 1/X + B -3.324342 4.355200 -0.8234856 Y = M * Xlog(X) + B 0.4316477E-02 72.83771 0.8834987 Ylog(Y) = M * X + B 0.1332904 309.7221 0.8854976 _____ (26) first by coefficient, fill by four-sort, NO backup _____ Correlation of panes vs. CPU time (in seconds) Equation:Slope (M):Intercept (B):Correlation Coeff.:Y = M * X + B0.1803455-11.225070.9739599log(Y) = M * X + B0.2070678E-01-0.54403000.9848391Y = M * log(X) + B18.98454-78.857140.9175950 Y = M * log(X) + B18.98454-78.85714log(Y) = M*log(X) + B2.348258-9.1097481/Y = M * log(X) + B-0.52040602.684677log(Y) = M * 1/X + B-221.90224.144630Y = M * Xlog(X) + B0.3136881E-01-7.893149Ylog(Y) = M * X + B0.6133339-45.893720.9995713 -0.9131927 -0.9782787 0.9802356 0.9484016 _____ Correlation of panes vs. efficiency Correlation of panes vs. efficiencyEquation :Slope (M) :Intercept (B) : Correlation Coeff. :Y = M * X + B0.3854412E-0161.075740.9195113log(Y) = M * X + B0.5882640E-034.1140750.9135152Y = M * log(X) + B4.31844245.381540.9220179log(Y) = M*log(X) + B0.6604662E-013.8738910.91792961/Y = M * log(X) + B-0.1010486E-020.1998901E-01-0.9171413log(Y) = M * 1/X + B-6.1432884.245754-0.8842857Y = M * Xlog(X) + B0.6633829E-0261.830980.9157143Ylog(Y) = M * X + B0.1997592251.02460.9203053 _____

(27) first by four-sort, fill by four-sort, NO backup _____ Correlation of panes vs. CPU time (in seconds) Equation: Slope (M): Intercept (B): Correlation Coeff.: Y = M * X + B 0.2243898 -15.06610 0.9637041 Log(Y) = M * X + B0.2332255E-01-15.06610Log(Y) = M * log(X) + B23.43312-98.32314log(Y) = M*log(X) + B2.641848-10.456481/Y = M * log(X) + B-0.61247723.138479log(Y) = M * 1/X + B-249.26654.451440Y = M * Xlog(X) + B0.3908021E-01-10.95136Ylog(Y) = M * X + B0.7969350-61.82316Correlation of paper 0.9860178 0.9007123 0.9996128 3.138479 4.451440 -0.9045013 -0.9768342 0.9711677 -61.82316 0.9342371 _____ Correlation of panes vs. efficiency Equation :Slope (M) :Intercept (B) :Correlation Coeff. :Y = M * X + B0.3735294E-0169.080880.9233541log(Y) = M * X + B0.5092773E-034.2366990.9246140Y = M * log(X) + B4.28238453.408960.947424253.40896 4.022570

 1
 M
 IOG(X) + B
 4.282384

 log(Y) =
 M*log(X) + B
 0.5848274E-01

 1/Y =
 M
 log(X) + B
 -0.7989779E-03

 log(Y) =
 M
 1/X + B
 -5.577499

 Y =
 M
 Xlog(X) + B
 0.6404987E-02

 Ylog(Y) =
 M
 X + B
 0.1977978

 0.9502748 0.1736321E-01 -0.9474012 4.353150 -0.9386289 69.82738 0.9161329 292.4742 0.9235295 (28) first by area, fill by four-sort, NO backup _____ Correlation of panes vs. CPU time (in seconds) Equation: Slope (M): Intercept (B): Correlation Coeff.: Y = M * X + B 0.1964589 -12.46611 0.9735942 $\begin{array}{cccccc} & & & & & & & & \\ log(Y) & = & M & * & X & + & B & & \\ 0.1964589 & & & & -12.46611 & \\ log(Y) & = & M & * & log(X) & + & B & & \\ 20.66114 & & & -86.04777 & \\ log(Y) & = & M^*log(X) & + & B & & 2.422328 & & -9.417209 & \\ 1/Y & = & M & * & log(X) & + & B & & 0 & \text{FORTUPE} \end{array}$ 0.9735942 0.9852764 0.9163806 0.9995899 $\begin{array}{l} 1/Y = M + \log(X) + B & -0.5225663 \\ \log(Y) = M + 1/X + B & -228.7014 \end{array}$ -0.9160876 2.691535 4.253352 -0.9774414Y = M * Xlog(X) + B0.3417537E-01-8.838855Ylog(Y) = M * X + B0.6806997-51.257310.9799780 0.9489622 _____ Correlation of panes vs. efficiency Equation : Slope (M) : Y = M * X + B 0.1919118E-01 Intercept (B) : Correlation Coeff. : 0.1919118E-01 75.19485 0.7633097 Y = M * X + B0.1919118E-01log(Y) = M * X + B0.2481043E-03Y = M * log(X) + B2.195787log(Y) = M*log(X) + B0.2837293E-011/Y = M * log(X) + B-0.3671192E-03log(Y) = M * 1/X + B-2.671430Y = M * X log(X) + B0.3288392E-024.320363 0.7756153 67.16391 4.216606 0.7816362

 -0.3671192E-03
 4.216606

 -2.671430
 4.376661

 0.3288392E-02
 75.57983

 0.1026452
 324

 0.7938373 -0.7824439 -0.7741125 Y = M * Xlog(X) + BYlog(Y) = M * X + B0.7567971 324.8262 0.7631264 _____

(29) first by longest side, fill by area, NO backup _____ Correlation of panes vs. CPU time (in seconds) Equation: Slope (M): Intercept (B): Correlation Coeff.: Y = M * X + B 0.1112368 -6.376969 0.9839500 Y = M * X + BStope (M)Intercept (B)Corre $\log(Y) = M * X + B$ 0.1112368-6.376969 $\log(Y) = M * \log(X) + B$ 0.1880307E-01-0.6524718 $Y = M * \log(X) + B$ 11.80991-48.56863 $\log(Y) = M*\log(X) + B$ 2.135029-8.443333 $1/Y = M * \log(X) + B$ -0.62040353.223217 $\log(Y) = M * 1/X + B$ -201.84083.608344Y = M * Xlog(X) + B0.1932058E-01-4.304923Ylog(Y) = M * X + B0.3310028-24.292940.9839500 0.9837264 0.9349459 0.9996885 -0.9333778 0.9888753 0.9612224 _____ Correlation of panes vs. efficiency Equation : Slope (M) : Intercept (B) : Correlation Coeff. : Y = M * X + B 0.4304412E-01 80.03823 0.9150031 log(Y) = M * X + B 0.5084444E-03 4.383682 0.9123676 Y = M * log(X) + B 5.051009 61.42680 0.9609532 log(Y) = M*log(X) + B 0.5981623E-01 4.163115 0.9606401 R1/Y = M * log(X) + B -0.7086097E-03 0.1508064E-01 -0.9563276 log(Y) = M * 1/X + B -5.889469 4.502982 -0.9796045 Y = M * Xlog(X) + B 0.7356591E-02 80.91333 0.9048616 Ylog(Y) = M * X + B 0.2341121 350.6693 0.9157804 _____ (30) first by coefficient, fill by area, NO backup Correlation of panes vs. CPU time (in seconds) Correlation of panes vs. CPU time (in seconds)Equation :Slope (M) : Intercept (B) : Correlation Coeff. :Y = M * X + B0.9574340E-01-5.5967360.9751828log(Y) = M * X + B0.1875615E-01-0.81498000.9865213Y = M * log(X) + B10.10170-41.611230.9208463log(Y) = M*log(X) + B2.123221-8.5556190.99947841/Y = M * log(X) + B-0.72498813.768580-0.9292713log(Y) = M * 1/X + B-200.37493.426095-0.9769099Y = M * Xlog(X) + B0.1664829E-01-3.8247650.9811684Ylog(Y) = M * X + B0.2712667-20.471460.9451893_____ Correlation of panes vs. efficiency Correlation of panes vs. efficiencyEquation :Slope (M) :Intercept (B) : Correlation Coeff. :Y = M * X + B0.5683777E-0165.976540.9385078log(Y) = M * X + B0.7799001E-034.1933520.9387289Y = M * log(X) + B6.27359943.282090.9271135log(Y) = M*log(X) + B0.8625056E-013.8811550.92913441/Y = M * log(X) + B-0.1186451E-020.1933907E-01-0.9297366log(Y) = M * 1/X + B-7.8706484.365335-0.8781325Y = M * Xlog(X) + B0.9803627E-0267.077200.9366651Ylog(Y) = M * X + B0.3006287276.10830.9384846 _____

(31) first by four-sort, fill by area, NO backup _____ Correlation of panes vs. CPU time (in seconds)
 0.10589/1
 -10.16839
 Correlation Coef

 0.2015287E-01
 -0.5259345
 0.9740379

 17
 46020
 0.9740379
 Equation : Slope (M) : Intercept (B) : Correlation Coeff. : Y = M * X + B log(Y) = M * X + B Y = M * log(X) + B 17.46939 -72.40965 0.9179731 1 - M10g(X) + B17.40939-72.40903log(Y) = M*log(X) + B2.284973-8.8602941/Y = M * log(X) + B-0.52246302.699969log(Y) = M * 1/X + B-215.96354.037278Y = M * Xlog(X) + B0.2885486E-01-7.102901Ylog(Y) = M * X + B0.5533714-41.26307-8.860294 0.9997163 2.699969 -0.9164727 4.037278 -0.9786085 0.9802858 -41.26307 0.9481351 ____ Correlation of panes vs. efficiency Correlation of panes vs. efficiencyEquation :Slope (M) :Intercept (B) : Correlation Coeff. :Y = M * X + B0.4202987E-0176.421260.8310285log(Y) = M * X + B0.5209961E-034.3372060.8262225Y = M * log(X) + B5.01586757.849960.8876039log(Y) = M * log(X) + B0.6230390E-014.1063910.88428691/Y = M * log(X) + B-0.7742245E-030.1593149E-01-0.8821926log(Y) = M * 1/X + B-6.1767504.460794-0.9079688Y = M * Xlog(X) + B0.2265698331.30690.8318136(32) first by area, fill by area, NO backup Correlation of panes vs. CPU time (in seconds) Equation: Slope (M): Intercept (B): Correlation Coeff.: Y = M * X + B 0.3833486 -26.50795 0.9603000 0.2472224E-01 39.91385 log(Y) = M * X + BY = M * log(X) + B -0.5478525 0.9865388 -168.1775 39.91385 2.798807 0.8948537 1 - M10g(X) + B39.91305-108.1775log(Y) = M*log(X) + B2.798807-10.751721/Y = M * log(X) + B-0.43521932.223824log(Y) = M * 1/X + B-263.83905.039674Y = M * Xlog(X) + B0.6679513E-01-19.49688Ylog(Y) = M * X + B1.554265-120.64670.9995732 -0.8992518 -0.9759187 0.9681767 0.9337559 _____ Correlation of panes vs. efficiency Equation : Slope (M) : Intercept (B) : Correlation Coeff. : 79.96391 4.382576 0.3633869E-01 0.9436247 Y = M * X + Blog(Y) = M * X + B Y = M * log(X) + B 0.4322151E-03 4.1853880.9430971 64.62593 4.199799 0.9727046 log(Y) = M*log(X) + B = 0.4985433E-011/Y = M*log(X) + B = -0.5942542E-03log(Y) = M*1/X + B = -4.7954340.9735869 0.1466140E-01 -0.9723352 log(Y) = M * 1/X + B -4.795434Y = M * Xlog(X) + B 0.6228250E-02Ylog(Y) = M * X + B 0.19738794.481992 -0.9699126 80.69186 0.9358209 350.2847 0.9440913 _____

<pre>(1) first by longest si fill by longest si with backup</pre>	ide, le,		
Correlation of panes vs. Equation : Y = M * X + B log(Y) = M * X + B	. CPU time (in se Slope (M) : 0.2452735 0.2225692E-01	conds) Intercept (B) : -16.03794 -0.5353718	Correlation Coeff. : 0.9693798 0.9873052
Y = M * log(X) + B log(Y) = M*log(X) +B l/Y = M * log(X) + B log(Y) = M * 1/X + B Y = M * Xlog(X) + B Ylog(Y) = M * X + B	25.70493 2.515548 -0.4645548 -236.5500 0.4269098E-01 0.8980303	-107.4754 -9.701941 2.390846 4.485698 -11.52409 -68.25776	0.9092328 0.9986982 -0.9273990 -0.9726519 0.9762842 0.9440653
Correlation of panes vs. Equation : Y = M * X + B log(Y) = M * X + B Y = M * log(X) + B log(Y) = M*log(X) + B log(Y) = M * log(X) + B log(Y) = M * 1/X + B Y = M * Xlog(X) + B Ylog(Y) = M * X + B	<pre>efficiency Slope (M) : 0.3469164E-01 0.4161305E-03 4.097714 0.4922681E-01 -0.5917373E-03 -4.838893 0.5918973E-02 0.1881452</pre>	<pre>Intercept (B) : 79.74479 4.379524 64.61738 4.197716 0.1470983E-01 4.477340 80.45628 349.1321</pre>	Correlation Coeff. : 0.8682390 0.8664180 0.9178495 0.9173074 -0.9126697 -0.9338824 0.8571517 0.8690737
<pre>(4) first by area, fill by longest sid with backup</pre>	le ,		
Correlation of panes vs. Equation : Y = M * X + B log(Y) = M * X + B Y = M * log(X) + B log(Y) = M*log(X) + B l/Y = M * log(X) + B log(Y) = M * 1/X + B Y = M * Xlog(X) + B Ylog(Y) = M * X + B	. CPU time (in se Slope (M) : 0.1103647 0.1751394E-01 11.73173 1.983460 -0.5039703 -186.9692 0.1916596E-01 0.3329233	<pre>conds) Intercept (B) : -6.003086 -0.4197966 -47.93245 -7.651856 2.636285 3.539117 -3.945358 -23.83563</pre>	Correlation Coeff. : 0.9842519 0.9858316 0.9363834 0.9992120 -0.9484265 -0.9755220 0.9890167 0.9626146
Correlation of panes vs. Equation : Y = M * X + B log(Y) = M * X + B Y = M * log(X) + B log(Y) = M*log(X) + B l/Y = M * log(X) + B log(Y) = M * 1/X + B Y = M * Xlog(X) + B Ylog(Y) = M * X + B	. efficiency Slope (M) : 0.4733824E-01 0.5812558E-03 5.550336 0.6832138E-01 -0.8418748E-03 -6.718871 0.8090608E-02 0.2556397	<pre>Intercept (B) : 76.35147 4.336881 55.90503 4.085016 0.1616526E-01 4.473133 77.31380 330.8865</pre>	Correlation Coeff. : 0.8890663 0.8855459 0.9329464 0.9315706 -0.9309787 -0.9488300 0.8792249 0.8896269

(13) first by longest side, fill by area, with backup _____ Correlation of panes vs. CPU time (in seconds) Equation : Slope (M) : Intercept (B) : Correlation Coeff. : -6.237420 0.9843903 Y = M * X + B0.1122044 -0.4916362 log(Y) = M * X + B0.1797527E-01 0.9857605 11.92125 Y = M * log(X) + B-48.83699 0.9360405 log(Y) = M*log(X) +B2.034889 -7.910313 0.9987391 1/Y = M * log(X) + B -0.5301112log(Y) = M * 1/X + B -191.6549 2.768237 -0.9484274 3.569294 -0.9742359 Y = M * Xlog(X) + B 0.1948631E-01 Ylog(Y) = M * X + B 0.3380831 -4.145920 0.9891995 0.9630864 -24.42667 _____

 Y = M * X + B
 0.2783824E-01
 Slope (M):
 Intercept (B):
 Correl

 log(Y) = M * X + B
 0.3311121E-03
 4.397317

 Y = M * log(X) + B
 3.320484
 68.90917

 log(Y) = M*log(X) + B
 0.3955625E-01
 4.250817

 l/Y = M * log(X) + B
 -0.4712976E-03
 0.1405209E-01

 log(Y) = M * 1/X + B
 -3.946240
 4.476058

 Y = M * Xlog(X) + B
 0.1512132
 257.05

 Intercept (B) : Correlation Coeff. : 0.7855093 0.7854290 0.8385450 0.8397738 -0.8396749 -0.8676880 0.7745110 0.7854960 _____ (16) first by area, fill by area, with backup Correlation of panes vs. CPU time (in seconds) Equation : Slope (M) : Intercept (B) : Correlation Coeff. : Y = M * X + B 0.2954749 -19.71499 0.9654933 -19.71499 0.9654933 log(Y) = M * X + B 0.2295126E-01Y = M * log(X) + B 30.88099-0.4831922 0.9878576 30.88099 2.593142 -129.4633 0.9030987 log(Y) = M*log(X) + B 2.593142 l/Y = M*log(X) + B -0.4281681 log(Y) = M*1/X + B -243.8158-9.931536 0.9989172 2.199434 -0.9222475 4.693441 -0.9727424 $Y = M * X \log(X) + B$ -243.81584.093441 $Y = M * X \log(X) + B$ 0.5145250E-01-14.29180 $Y \log(Y) = M * X + B$ 1.132151-86.618570.9728186 0.9395676 _____ Correlation of panes vs. efficiency Equation: Slope (M): Y = M * X + B 0 4450000F-01 Intercept (B) : Correlation Coeff. : EquationStope (M)Intercept (B)Correlation CoeffY = M * X + B0.4450000E-0178.550000.9219491 $\log(Y) = M * X + B$ 0.5332318E-034.3651460.9191045 $Y = M * \log(X) + B$ 5.15033659.648750.9549878 $\log(Y) = M*\log(X) + B$ 0.6183326E-014.1380960.9538636 $1/Y = M * \log(X) + B$ -0.7428397E-030.1542581E-01-0.9503824 $\log(Y) = M * 1/X + B$ -5.9531284.488146-0.9511353Y = M * Xlog(X) + B0.7618210E-0279.446860.9132646Ylog(Y) = M * X + B0.2413952342.65490.9225714

(17) first by longest side, fill by longest side, NO backup Correlation of panes vs. CPU time (in seconds) Equation :Slope (M) :Intercept (B) :Correlation Coeff. :Y = M * X + B0.2382691-15.934270.9681892log(Y) = M * X + B0.2350570E-01-0.78538420.9845269Y = M * log(X) + B24.94388-104.63240.9071349log(Y) = M*log(X) + B2.666433-10.512550.99954061/Y = M * log(X) + B-0.58887203.015741-0.9054677log(Y) = M * 1/X + B-251.82654.536376-0.9776964Y = M * Xlog(X) + B0.4147888E-01-11.553630.9752506Ylog(Y) = M * X + B0.8586992-66.150600.9414378_____ Correlation of panes vs. efficiency Correlation of panes vs. efficiencyEquation :Slope (M) :Y = M * X + B0.3608824E-01log(Y) = M * X + B0.4419807E-034.3538140.9166803Y = M * log(X) + B4.21676462.183160.9599053log(Y) = M*log(X) + B0.5174809E-011/Y = M * log(X) + B-0.6352690E-03log(Y) = M * 1/X + B-5.052313Y = M * Xlog(X) + B0.6169886E-02Y = M * Xlog(X) + B0.1949706338.14890.9182752 _____ (20) first by area, fill by longest side, NO backup _____ Correlation of panes vs. CPU time (in seconds) Equation:Slope (M):Intercept (B):Correlation Coeff.:Y = M * X + B0.1026868-5.8654680.9837601log(Y) = M * X + B0.1869583E-01-0.71281460.9837340Y = M * log(X) + B10.90123-44.809750.9346860

 Y = M * log(X) + B
 10.90123
 -44.80975

 log(Y) = M*log(X) + B
 2.122820
 -8.459090

 1/Y = M * log(X) + B
 -0.6623291
 3.442253

 log(Y) = M * 1/X + B
 -200.7133
 3.523923

 Y = M * Xlog(X) + B
 0.1783593E-01
 -3.952924

 Ylog(Y) = M * X + B
 0.2977989
 -21.95566

 0.9996812 -0.9331838 -0.9789427 0.9887061 0.9602202 _____ Correlation of panes vs. efficiency

 Correlation of panes vs. efficiency

 Equation :
 Slope (M) :

 Y = M * X + B
 0.4966130E-01

 log(Y) = M * X + B
 0.6122185E-03

 Y = M * log(X) + B
 5.796683

 10g(Y) = M*log(X) + B
 0.7164948E-01

 4.065209
 0.9359270

 1/Y = M * log(X) + B
 -0.8865226E-03

 10g(Y) = M * 1/X + B
 -7.013701

 4.471924
 -0.9488753

 Y = M * Xlog(X) + B
 0.2679908

(29) first by longest side, fill by area, NO backup _____ Correlation of panes vs. CPU time (in seconds) Equation : Slope (M) : Intercept (B) : Correlation Coeff. :

 0.1083316
 -6.219262

 0.1884973E-01
 -0.6874398

 11.50332
 -47.31779

 2.139797
 -8 405115

 0.9842530 Y = M * X + Bloq(Y) = M * X + B0.9837834 Y = M * log(X) + B0.9353842 1 - M = 10g(X) + B11.30332-47.31779log(Y) = M*log(X) + B2.139797-8.4951161/Y = M * log(X) + B-0.63905973.320321log(Y) = M * 1/X + B-202.19263.582538Y = M * Xlog(X) + B0.1881527E-01-4.200894Ylog(Y) = M * X + B0.3192264-23.483740.9994988 -0.9352461 3.582538 -0.9781554 0.9891424 0.9615971 _____ Correlation of panes vs. efficiency Equation : Slope (M) : Y = M * X + B 0.3480836E-01 Intercept (B) : Correlation Coeff. : Y = M * X + B0.3480836E-0179.85522 $\log(Y) = M * X + B$ 0.4162167E-034.380983 $Y = M * \log(X) + B$ 4.09251564.76709 $\log(Y) = M*\log(X) + B$ 0.4903631E-014.200091 $1/Y = M * \log(X) + B$ -0.5876009E-030.1467287E-01 $\log(Y) = M * 1/X + B$ -4.8197144.478629 $Y = M * X\log(X) + B$ 0.1888860349.720079.85522 0.8409032 0.8423634 0.8848466 0.8882048 0.1467287E-01 -0.8859811 -0.9041688 0.8310801 0.8409606 _____ (32) first by area, fill by area, NO backup Correlation of panes vs. CPU time (in seconds) Equation: Slope (M): Intercept (B): Correlation Coeff.: Y = M * X + B 0.2870663 -19.43141 0.9659825 Y = M * X + B0.2870005log(Y) = M * X + B0.2393621E-01-0.6831431Y = M * log(X) + B30.00000-126.0461log(Y) = M*log(X) + B2.712674-10.57613-19.43141 0.9659825 0.9854137 0.9034907 -10.57613 0.9994846 1/Y = M * log(X) + B -0.5176718log(Y) = M * 1/X + B -255.9297 -0.9052964 2.649361 -0.9766347 Y = M * Xlog(X) + BYlog(Y) = M * X + B0.9733010 -83.71247 0.9397853 _____ Correlation of panes vs. efficiency Equation: Slope (M): Intercept (B): Correlation Coeff.: Y = M * X + B 0.4251471E 01 772 20041 EquationSlope (M)Intercept (B)Correlation coeffY = M * X + B0.4351471E-0177.329410.9285323 $\log(Y) = M * X + B$ 0.5295554E-034.3495110.9239219 $Y = M * \log(X) + B$ 5.02178458.915620.9590347 $\log(Y) = M*\log(X) + B$ 0.6123935E-014.1248230.9562464 $1/Y = M * \log(X) + B$ -0.7470637E-030.1563877E-01-0.9559872 $\log(Y) = M * 1/X + B$ -5.8908974.471463-0.9526944Y = M * Xlog(X) + B0.7453881E-0278.203760.9203229Ylog(Y) = M * X + B0.2353732336.11780.9292206_____

DATA SET THREE

(1) first by longest a fill by longest a with backup	side, ide,		
Correlation of panes vi Equation : Y = M * X + B log(Y) = M * X + B Y = M * log(X) + B log(Y) = M*log(X) + B l/Y = M * log(X) + B log(Y) = M * 1/X + B Y = M * Xlog(X) + B Ylog(Y) = M * X + B	s. CPU time (in se Slope (M) : 0.2588014 0.2207192E-01 27.11259 2.491422 -0.4252805 -233.9303 0.4504814E-01 0.9625570	conds) Intercept (B) : (-16.89580 -0.4517081 -113.3286 -9.526805 2.191144 4.521441 -12.13457 -72.98405	Correlation Coeff. : 0.9687365 0.9883523 0.9082919 0.9984683 -0.9331654 -0.9709712 0.9756920 0.9439018
Correlation of panes v Equation : Y = M * X + B log(Y) = M * X + B Y = M * log(X) + B log(Y) = M*log(X) + B log(Y) = M * log(X) + B log(Y) = M * 1/X + B Y = M * Xlog(X) + B Ylog(Y) = M * X + B 	<pre>s. efficiency Slope (M) : 0.3657353E-01 0.4381319E-03 4.285970 0.5145675E-01 -0.6175193E-03 -5.040276 0.6250706E-02 0.1984026</pre>	<pre>Intercept (B) : (79.53455 4.377057 63.74818 4.187407 0.1482905E-01 4.479530 80.27811 347.9868</pre>	Correlation Coeff. : 0.8903177 0.8834745 0.9337758 0.9286388 -0.9319982 -0.9420885 0.8804485 0.8907614
<pre>====================================</pre>	<pre>s. CPU time (in se Slope (M) : 0.1130824 0.1753963E-01 12.01806 1.988441 -0.4980579 -187.7111 0.1963923E-01 0.3440015</pre>	<pre>====================================</pre>	Correlation Coeff. : 0.9833514 0.9848419 0.9353274 0.9992501 -0.9444227 -0.9769768 0.9881776 0.9609364
Correlation of panes v Equation : Y = M * X + B log(Y) = M * X + B Y = M * log(X) + B log(Y) = M*log(X) + B log(Y) = M * log(X) + B log(Y) = M * 1/X + B Y = M * Xlog(X) + B Ylog(Y) = M * X + B	<pre>s. efficiency Slope (M) : 0.4692601E-01 0.5747357E-03 5.428776 0.6667414E-01 -0.8190037E-03 -6.466276 0.8038174E-02 0.2535349</pre>	<pre>Intercept (B) : 0 76.42800 4.338031 56.50743 4.093175 0.1605214E-01 4.471076 77.37092 331.2823</pre>	Correlation Coeff. : 0.9114054 0.9073585 0.9436594 0.9420705 -0.9424145 -0.9462658 0.9033419 0.9118063

(13) first by longest side, fill by area, with backup _____ Correlation of panes vs. CPU time (in seconds) Equation : Slope (M) : Intercept (B) : Correlation Coeff. : -6.833457 0.9846426 Y = M * X + B0.1238427 -0.3699834 log(Y) = M * X + B0.1786633E-01 0.9850340 13.16569 Y = M * log(X) + B-53.88926 0.9368439 log(Y) = M*log(X) +B2.023644 -7.748866 0.9985396 1/Y = M * log(X) + B -0.4724328log(Y) = M * 1/X + B -190.6836 2.467918 -0.9489894 3.668136 -0.9744899 Y = M * Xlog(X) + B 0.2150576E-01 Ylog(Y) = M * X + B 0.3860026 -4.523950 0.9893727 -27.56975 0.9640369 _____ -----

 Y = M * X + B
 0.3361811E-01
 82.04773

 log(Y) = M * X + B
 0.3923771E-03
 4.408002

 Y = M * log(X) + B
 3.937965
 67.54492

 log(Y) = M * log(X) + B
 0.4604438E-01
 4.238342

 l/Y = M * log(X) + B
 -0.5386414E-03
 0.1415741E-01

 log(Y) = M * 1/X + B
 -4.526311
 4.499891

 Y = M * Xlog(X) + B
 0.1832408
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 Intercept (B) : Correlation Coeff. : 0.9019524 0.9016267 0.9455785 0.9469240 -0.9426699 -0.9640862 0.8922074 0.9024568 _____ (16) first by area, fill by area, with backup Correlation of panes vs. CPU time (in seconds)

 Equation :
 Slope (M) :
 Intercept (B) :
 Correlation Coeff. :

 Y = M * X + B
 0.3270133
 -21.85104
 0.9639017

 -21.85104 0.9639017 log(Y) = M * X + B 0.2292149E-01Y = M * log(X) + B 34.15173-0.3793676 0.9881976 34.15173 2.588430 -143.1928 0.9009388 log(Y) = M*log(X) + B = 2.588430 l/Y = M*log(X) + B = -0.3842843 log(Y) = M*1/X + B = -243.2224-9.809052 0.9987408 1.974695 -0.9252954 4.787928 -0.9719700 $Y = M * X \log(X) + B$ -243.22244.787928 $Y = M * X \log(X) + B$ 0.5695236E-01-15.85385 $Y \log(Y) = M * X + B$ 1.286821-98.280190.9713503 0.9378192 _____ Correlation of panes vs. efficiency Equation: Slope (M): Intercept (B): Correlation Coeff.: Y = M * X + B 0.4216177F-01 79.56104 0.0455401 EquationStope (M):Intercept (B):Correlation CoeffY = M * X + B0.4216177E-0178.561040.9455401 $\log(Y) = M * X + B$ 0.5055147E-034.3653130.9447196 $Y = M * \log(X) + B$ 4.81219260.973670.9658716 $\log(Y) = M*\log(X) + B$ 0.5781040E-014.1539070.9669197 $1/Y = M * \log(X) + B$ -0.6946463E-030.1523553E-01-0.9650775 $\log(Y) = M * 1/X + B$ -5.4932334.480496-0.9515802Y = M * Xlog(X) + B0.7235417E-0279.400050.9389046Ylog(Y) = M * X + B0.2286820342.71240.9458404342.7124 0.945015 -----

(17) first by longest side, fill by longest side, NO backup Correlation of panes vs. CPU time (in seconds) Equation :Slope (M) : Intercept (B) : Correlation Coeff. :Y = M * X + B0.2426412-16.413900.9677447log(Y) = M * X + B0.2403514E-01-0.86603980.9851916Y = M * log(X) + B25.38333-106.65280.9060667log(Y) = M*log(X) + B2.723863-10.799810.99925091/Y = M * log(X) + B-0.61668883.155985-0.9067762log(Y) = M * 1/X + B-256.90524.569971-0.9761018Y = M * Xlog(X) + B0.8752130-67.798730.9414330_____ Correlation of panes vs. efficiency Correlation of panes vs. efficiencyEquation :Slope (M) :Y = M * X + B0.3447059E-01log(Y) = M * X + B0.4274471E-034.3443230.8987659Y = M * log(X) + B4.062214log(Y) = M*log(X) + B0.5048185E-014.1579690.94998071/Y = M * log(X) + B-0.6275651E-03log(Y) = M * 1/X + B-4.986002Y = M * Xlog(X) + B0.5886470E-02Y = M * Xlog(X) + B0.1858051334.29540.9035569 _____ (20) first by area, fill by longest side, NO backup _____ Correlation of panes vs. CPU time (in seconds) Equation:Slope (M):Intercept (B):Correlation Coeff.:Y = M * X + B0.1038786-5.9566430.9834729log(Y) = M * X + B0.1877405E-01-0.71676490.9838488Y = M * log(X) + B11.02438-45.336890.9341267 Y = M * log(X) + B11.02438-45.33689log(Y) = M*log(X) + B2.131533-8.4946471/Y = M * log(X) + B-0.66304423.444873log(Y) = M * 1/X + B-201.52933.537474Y = M * Xlog(X) + B0.1804383E-01-4.022438Ylog(Y) = M * X + B0.3021569-22.303800.9997187 -0.9324306 -0.9789419 0.9884658 0.9598647 _____ Correlation of panes vs. efficiency Correlation of panes vs. efficiencyEquation :Slope (M) :Y = M * X + B0.4504412E-01log(Y) = M * X + B0.5578182E-03Y = M * log(X) + B5.219902log(Y) = M*log(X) + B0.6482519E-014.0909980.95755101/Y = M * log(X) + B-0.8050622E-03log(Y) = M * 1/X + B-6.292030Y = M * Xlog(X) + B0.7712866E-02Y = M * Xlog(X) + B0.2428621327.74880.9265124 -----

<pre>(29) first by longest sid fill by area, NO backup</pre>	de,		
		========================	
Correlation of panes vs.	CPU time (in se	conds)	
Equation :	Slope (M) :	Intercept (B) :	Correlation Coeff. :
Y = M * X + B	0.1137956	-6.566576	0.9845852
loq(Y) = M * X + B	0.1906300E-01	-0.6751937	0.9827476
Y = M * log(X) + B	12 08765	-49 75766	0 9360198
$\log(\mathbf{X}) = M^* \log(\mathbf{X}) + B$	2 166231	-8 581773	0 9994729
1/Y = M * log(X) + P	-0.6307471	3 273202	
1/1 = M 10g(X) + B		2 647020	-0.9324319
$\log(Y) = M + 1/X + B$	-204.8960	3.647029	-0.9/91124
Y = M * Xlog(X) + B	0.1976311E-01	-4.445697	0.9894183
Ylog(Y) = M * X + B	0.3401138	-24.97748	0.9623000
Correlation of panes vs.	efficiency		
Equation :	Slope (M):	Intercept (B) :	Correlation Coeff. :
Y = M * X + B	0 3202941E = 01	79 73383	0 8798699
$\log(V) - M * V + P$	0.3850528E - 03	1 270252	0 8765275
$X = M + \log(X) + D$	2 7 5 5 0 9 7	4.379332 6E 00100	0.0705275
$Y = M ^ IOg(X) + B$	3.755087	65.90109	0.9232193
log(Y) = M*log(X) + B	0.4523/56E-01	4.212608	0.9216367
1/Y = M * log(X) + B	-0.5450541E-03	0.1453594E-01	-0.9209613
log(Y) = M * 1/X + B	-4.450410	4.469607	-0.9390604
Y = M * Xlog(X) + B	0.5474686E-02	80.38463	0.8702129
Ylog(Y) = M * X + B	0.1736379	349.0756	0.8803290
fill by area, NO backup			
Correlation of paper wa	CDU time (in ge		
Equation :	CPO CIME (III Second	Theorem (D)	Correlation Cooff .
	STOPE (M) ·		
$X = M \wedge X + B$	0.2962515	-20.06956	0.9656370
LOG(Y) = M * X + B	0.2399981E-01	-0.6624224	0.9850622
Y = M * Log(X) + B	30.95589	-130.0766	0.9030505
log(Y) = M*log(X) + B	2.720860	-10.58635	0.9994873
1/Y = M * log(X) + B	-0.5072491	2.595385	-0.9047503
log(Y) = M * 1/X + B	-256.7944	4.768124	-0.9769890
Y = M * Xlog(X) + B	0.5158872E-01	-14.63273	0.9729820
Ylog(Y) = M * X + B	1.129035	-87.06079	0.9392701
Correlation of papes vg	efficiency		
Equation :	Slope(M)	Intercent (B) ·	Correlation Coeff .
$\mathbf{\nabla} = \mathbf{M} + \mathbf{\nabla} \mathbf{P}$		$\frac{1}{76} 0 2 5 2 6$	
$I = M \cdot X + B$	0.3911/19E-U1	10.03530	0.9415929
$TOQ(X) = M \circ X + B$	U.4015U20E-U3	4.342895	0.9405282
X = M * LOG(X) + B	4.514/4/	60.2802/	0.9/26225
LOG(Y) = M*LOG(X) + B	U.5569252E-01	4.138546	0.9736103
I/Y = M * Log(X) + B	-0.6873363E-03	U.1550780E-01	-0.9712463
log(Y) = M * 1/X + B	-5.376321	4.453969	-0.9734339
Y = M * Xlog(X) + B	0.6702514E-02	77.62017	0.9335340
Ylog(Y) = M * X + B	0.2111434	333.4942	0.9419100
1109(1) - M ~ A + B	U.ZIII434	ی۲۶۵.494۵ 	0.9419100

APPENDIX F: CPU time vs. number of panes graphs

Graphs for DATA SET ONE









Graphs for DATA SET TWO


Graphs for DATA SET THREE



APPENDIX G: efficiency vs. number of panes graphs

Graphs for DATA SET ONE













Graphs for DATA SET TWO





Graphs for DATA SET THREE

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